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Christopher Chong · Panayotis G. Kevrekidis

Coherent Structures in Granular Crystals

From Experiment and Modelling
to Computation and Mathematical Analysis

 Springer

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To my family for their continued support: For my parents and sister, who helped me in an uncountable number of ways throughout my life. For my wife, who has given me an unwavering support and so much happiness and guidance since we met. And for my children, who give me inspiration everyday.

Christopher Chong

To my father, George, for wanting (and expecting) this book. This one is for you, dad.

Panayotis G. Kevrekidis

Preface

The topic of granular crystals has enjoyed a rich history over the past 30 years, starting with the pioneering work of V. F. Nesterenko and his collaborators in the 1980s. A number of significant developments both physically and mathematically took place in the 1990s, especially as regards the prototypical excitation of such chains of beads, namely, traveling waves. The early 2000s led to a further booming of interest fueled to a large degree by the development of novel experimental techniques that could more controllably and reliably provide quantitative information about the system (via suitable piezosensors). The 2001 authoritative book by Nesterenko constitutes, still to this day, an excellent summary of the early stages of the research on granular crystals. A 2008 Physics Reports review article by S. Sen, another of the key contributors in this field, gave a concise summary of the later developments. Around 2008, a new wave of contributions on the subject came along providing a deeper understanding of different types of excitations (such as discrete breathers and shock waves), and different types of setups (including heterogeneous and random ones, ones involving mass-in-mass, mass-with-mass, branching, intruder-based, and other types of configurations). Novel, noninvasive experimental techniques (such as laser Doppler vibrometry) also allowed researchers to probe granular crystals in unprecedented ways.

As this decade of significant and rapid developments is coming to a close, many of the newer topics are becoming more well understood. Consequently, we felt there was an opportunity to provide a brief, somewhat more mathematically minded (with a broad array of computational touches) summary of a select, coherent set of these developments, what motivated them, as well as where they stand presently from the point of view of theory, computation, and experiment. In that vein, there are two main goals of the present volume: One is to offer novices in granular crystals (but also to some degree more seasoned researchers) a reference point for the current state of the art (admittedly, with a considerable touch of personal flavor). The other goal is to offer some insights and hopefully pave the way toward some of the important future developments on this theme. We thus hope that it will be of

use to advanced undergraduates, graduate students, and young researchers, but also to our colleagues interested in the subject of nonlinear coherent structures and their applications in materials science more broadly. Finally, we would much welcome their thoughts and comments toward further improving its exposition in the future.

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