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Pairwise Comparison Matrices and their Fuzzy Extension

Multi-criteria Decision Making with a New Fuzzy Approach

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To my dad who has always lived for me and my sister, believed in us, and supported us in anything we chose to do.

Preface

Research on multi-criteria decision making (MCDM) methods based on pairwise comparison matrices (PCM) has been emerging rapidly since the introduction of the first well-known pairwise comparison (PC) methods in 1970s. Moreover, since the original PC methods were not designed to cope with large-dimensional PC problems and with uncertainty present in decision problems, the PC methods were also very soon extended to incomplete PCMs and fuzzy PCMs.

Hundreds of research papers on incomplete PC methods and fuzzy PC methods have been published; many of the methods base on the methods originally developed for PCMs. Moreover, many research papers criticizing and revealing mistakes in some fuzzy and incomplete PC methods have been published too. The research papers are usually very specialized and focused on one specific topic assuming a certain level of readers' knowledge (as expected from research papers). Naturally, also the notation used in the research papers varies from paper to paper. Thus, it is for most readers not simple to compare methods proposed in different research papers and identify relations among them.

Despite the huge number of the methods developed, no book providing a critical overview of the existing methods and presenting the research results in a broader and unified frame has been published so far. This book is the first step to close this gap. In particular, the book presents a detailed critical literature review of the fuzzy PC methods based on the fuzzy extension of the methods originally developed for PCMs. Differences and relations between various methods are emphasized, and the reviewed fuzzy PC methods and their drawbacks are illustrated by many numerical examples. In fact, it is shown in the book that the majority of the reviewed fuzzy PC methods violate the basic assumptions made on the original PC methods on which they are build. Thus, besides the literature review, the book also introduces constrained fuzzy arithmetic in fuzzy extension of PC methods in order to preserve the original assumptions. Based on constrained fuzzy arithmetic, new fuzzy PC methods and incomplete PC methods are introduced and critically compared with the reviewed methods.

The book is intended for researchers in MCDM as well as for graduate and Ph.D. students, in particular for those interested in AHP and other PC methods. The book is self-contained. Chapters 2 and 3 provide an overview of the concepts necessary for studying Chaps. 4 and 5. In particular, Chap. 2 provides a detailed critical overview of PC methods based on three well-known types of PCMs, while Chap. 3 provides an overview of the concepts from fuzzy set theory indispensable for the fuzzy extension of PC methods and a detailed introduction to constrained fuzzy arithmetic. Chapter 4 is then focused on fuzzy PC methods and Chap. 5 on incomplete PC methods. The readers with a solid background in PC methods and fuzzy set theory may wish to skip Chap. 2 and Sects. 3.1–3.3 in Chap. 3. Sections 3.4 and 3.5 are, however, highly recommended to all readers in order to fully understand the difference between standard and constrained fuzzy arithmetic, which plays a key role in Chap. 4. Actually, Sect. 3.5 on constrained fuzzy arithmetic is very interesting by itself, in particular then the examples provided in the section.

The book is suitable also for the readers interested only in studying standard PC methods, in particular then for graduate and Ph.D. students. These readers may read only Chap. 2 that provides a critical overview of the most-known PC methods developed for three different types of PCMs and shows relations between various methods. Further, the book is of value also for the researchers interested in studying constrained fuzzy arithmetic and applying it (not only) in MCDM. These readers may read only Example 1 on p. 9 and Sects. 3.4 and 3.5, or alternatively whole Chap. 3 in case they need to refresh the basics of fuzzy set theory.

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Acknowledgments

This book was created as the extension of my Ph.D. thesis written within my 3-year double-degree Ph.D. studies at the Department of Industrial Engineering at the University of Trento and at the Faculty of Business, Economics and Law at the University of Bayreuth. It summarizes the research results achieved during my Ph.D. studies and published in a number of research papers acknowledged in the book.

I would like to acknowledge the significant contribution of my Ph.D. supervisors, professors Michele Fedrizzi and Johannes Siebert, to the research results published in this book as well as to the organization of the book. Further, I would like to acknowledge the research contribution of all co-authors of my research papers on which this book is based. Last but not least, I would like to thank professors Janusz Kacprzyk, José Luis García Lapresta, and Jörg Schlüchtermann for contributing to increasing the quality of the book, as they acted as evaluators of my Ph.D. thesis.

Contents

Abbreviations

AHP	Analytic hierarchy process
APCM	Additive pairwise comparison matrix
APCM-A	Additive pairwise comparison matrix with additive representation
APCM-M	Additive pairwise comparison matrix with multiplicative representation
DM	Decision-maker
EVM	Eigenvector method
FAPCM	Fuzzy additive pairwise comparison matrix
FAPCM-A	Fuzzy additive pairwise comparison matrix with additive representation
FAPCM-M	Fuzzy additive pairwise comparison matrix with multiplicative representation
FMPCM	Fuzzy multiplicative pairwise comparison matrix
FPCM	Fuzzy pairwise comparison matrix
GMM	Geometric mean method
LLSM	Logarithmic least squares method
MCDM	Multi-criteria decision making
MPCM	Multiplicative pairwise comparison matrix
PC	Pairwise comparison
PCM	Pairwise comparison matrix

Mathematical Symbols

$\emptyset$	Empty set
$\mathbb{N}$	Set of natural numbers
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^+$	Set of positive real numbers greater than 0
$U \times V$	Cartesian product of two sets U and V
$\mathbb{R}^n$	$n-$ary cartesian power of set $\mathbb{R}$
$\mathcal{F}(\mathbb{R})$	Set of all fuzzy sets defined on $\mathbb{R}$
$\mathcal{F}_N(\mathbb{R})$	Set of all fuzzy numbers defined on $\mathbb{R}$
$\mathcal{F}_N(\mathbb{R}^+)$	Set of all positive fuzzy numbers
$\mathcal{F}_N(\mathbb{R})^n$	$n-$ary cartesian power of set $\mathcal{F}_N(\mathbb{R})$
$x \in \Omega$	Element belonging to set Ω
$\lvert\Omega\rvert$	Cardinality of set Ω
$\Omega \backslash Q$	Difference of sets Ω and Q
$\Omega \cap Q$	Intersection of sets Ω and Q
$\Omega \cup Q$	Union of sets Ω and Q
$(x_1, x_2, \ldots, x_n)$	$n-$tuple, i.e., an ordered list of n elements, $n \in \mathbb{N}$
$[a,b],\ \overline{c} = [c^L, c^U]$	Closed interval
$]a,b[$	Open interval
$\widetilde{c}$	Fuzzy set
$Supp\,\widetilde{c}$	Support of $\widetilde{c}$
$Core\,\widetilde{c}$	Core of $\widetilde{c}$
$\widetilde{c}_{(\alpha)}$	$\alpha-$cut of $\widetilde{c}$
$\widetilde{c}_{(0)} = Cl(Supp\,\widetilde{c})$	Closure of the support of $\widetilde{c}$
$c \in \widetilde{c}$	Element belonging to the closure of the support of $\widetilde{c}$
$\widetilde{c} = (c^L, c^M, c^U)$	Triangular fuzzy number
$\widetilde{c} = (c^\alpha, c^\beta, c^\gamma, c^\delta)$	Trapezoidal fuzzy number
$A = \{a_{ij}\}_{i,j=1}^n$	Square matrix
$\widetilde{A} = \{\widetilde{a}_{ij}\}_{i,j=1}^n$	Square fuzzy matrix
$\lvert A\rvert$	Determinant of matrix A

A^T	Transpose of matrix A
$\widetilde{A}^T$	Transpose of fuzzy matrix A
$\lambda = EVM_{\lambda}(A)$	Maximal eigenvalue of matrix A
$\underline{w} = EVM_{\underline{w}}(A)$	Normalized maximal eigenvector of matrix A
$\underline{w} = (w_1, \ldots, w_n)^T$	Column vector
$\underline{w}^T = (w_1, \ldots, w_n)$	Row vector
$\underline{\widetilde{w}} = (\widetilde{w}_1, \ldots, \widetilde{w}_n)^T$	Column fuzzy vector
$\underline{\overline{w}} = (\overline{w}_1, \ldots, \overline{w}_n)^T$	Column interval vector
$\wedge$	Logical conjunction
$\vee$	Logical disjunction
ln	Natural logarithm
$\log_9$	Logarithm of base 9
f^{-1}	Inverse of function f
arg f	Argument of function f
$\lfloor x \rfloor$	Floor of $x \in \mathbb{R}$
$k!$	Factorial of number $k \in \mathbb{N}$

Summary

Pairwise comparison (PC) methods form a significant part of multi-criteria decision making (MCDM) methods. PC methods are based on structuring PCs of objects from a finite set of objects into a pairwise comparison matrix (PCM) and deriving priorities of objects that represent the relative importance of each object with respect to all other objects in the set. However, crisp PCMs are not able to capture uncertainty stemming from subjectivity of human thinking and from incompleteness of information about the problem that are often closely related to MCDM problems. That is why the fuzzy extension of PC methods has been of great interest.

In order to derive fuzzy priorities of objects from a fuzzy PCM (FPCM), the fuzzy extension based on standard fuzzy arithmetic is usually applied to the methods originally developed for crisp PCMs. However, such approach fails in properly handling uncertainty of preference information contained in the FPCM. Namely, reciprocity of the related PCs of objects in a FPCM and invariance of the given method under permutation of objects are violated when standard fuzzy arithmetic is applied to the fuzzy extension. This leads to distortion of the preference information contained in the FPCM and consequently to false results. This issue is a motivation to the first research question dealt with in this book: *"Based on a FPCM of objects, how should fuzzy priorities of these objects be determined so that they reflect properly all preference information available in the FPCM?"* This research question is answered by introducing an appropriate fuzzy extension of PC methods originally developed for crisp PCMs, i.e., such fuzzy extension that does not violate the reciprocity of the related PCs and invariance of PC methods under permutation of objects, and that does not lead to a redundant increase of uncertainty of the resulting fuzzy priorities of objects.

Fuzzy extension of three well-known types of PCMs—multiplicative PCMs, additive PCMs with additive representation, and additive PCMs with multiplicative representation—is examined in this book. In particular, construction of PCMs, verifying consistency, and deriving priorities of objects from PCMs are studied in detail for each type of these PCMs. First, well-known and in practice most often applied PC methods based on crisp PCMs are reviewed. Afterwards, fuzzy extensions of these methods proposed in the literature are reviewed in detail, and

their drawbacks regarding the violation of reciprocity of the related PCs and of invariance of methods under permutation of objects are pointed out. It is shown that these drawbacks can be overcome by properly applying constrained fuzzy arithmetic to the computations instead of standard fuzzy arithmetic. In particular, we always have to look at a FPCM as a set of PCMs with different degrees of membership to the FPCM, i.e., we always have to consider only PCs that are mutually reciprocal. Constrained fuzzy arithmetic allows us to impose the reciprocity of the related PCs as a constraint on arithmetic operations with fuzzy numbers, and its appropriate application also guarantees invariance of the fuzzy PC methods under permutation of objects. Finally, new fuzzy extensions of the PC methods are proposed based on constrained fuzzy arithmetic, and it is proved that these methods do not violate the reciprocity of the related PCs and are invariant under permutation of objects. Because of these desirable properties, fuzzy priorities of objects obtained by the fuzzy PC methods proposed in this book reflect the preference information contained in FPCMs better in comparison to the fuzzy priorities obtained by the fuzzy PC methods based on standard fuzzy arithmetic.

Besides the inability to capture subjectivity, the PC methods are also not able to cope with situations where it is not possible or reasonable to obtain complete preference information from DMs. This problem occurs especially in the situations involving large-dimensional PCMs. When dealing with incomplete large-dimensional PCMs, a compromise between reducing the number of PCs required from the DM and obtaining reasonable priorities of objects is of paramount importance. This leads to the second research question: *"How can the amount of preference information required from the DM in a large-dimensional PCM be reduced while still obtaining comparable priorities of objects?"* This research question is answered by introducing an efficient two-phase PC method. Specifically, in the first phase, an interactive algorithm based on the weak-consistency condition is introduced for partially filling an incomplete PCM. This algorithm is designed in such a way that it minimizes the number of PCs required from the DM and provides a sufficient amount of preference information at the same time. The weak-consistency condition allows for providing ranges of possible intensities of preference for every missing PC in the incomplete PCM. Thus, at the end of the first phase, a PCM containing intervals for all PCs that were not provided by the DM is obtained. Afterward, in the second phase, the methods for obtaining fuzzy priorities of objects from FPCMs proposed in this book within the answer to the first research question are applied to derive interval priorities of objects from the incomplete PCM. The obtained interval priorities cover the priorities obtainable from all weakly consistent completions of the incomplete PCM and are very narrow. The performance of the method is illustrated by a real-life case study and by simulations that demonstrate the ability of the method to reduce the number of PCs required from the DM in PCMs of dimension 15 and greater by more than 60% on average while obtaining interval priorities comparable with the priorities obtainable from the hypothetical complete PCMs.