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How We Understand Mathematics

Conceptual Integration in the Language
of Mathematical Description

 Springer

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