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Andreas Pott

# Cable-Driven Parallel Robots

Theory and Application

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Andreas Pott  
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*“Theoria cum praxi”*  
*(Unity of theory and application)*  
*Maxim of Gottfried W. Leibniz*

*Dedicated to  
Eva and Jana.*

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Sindelfingen, Germany  
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Andreas Pott

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# Symbols

$\mathbf{A}^T$	Pose-dependent structure (wrench) matrix $\in \mathbb{R}^{m \times n}$
$\hat{\mathbf{A}}^T$	Non-normalized structure matrix $\in \mathbb{R}^{m \times n}$
$\mathbf{A}^{+T}$	Moore–Penrose pseudo-inverse of $\mathbf{A}^T$
$A_C$	Cross section of the cable
$\mathbf{a}_i$	Position vector $\in \mathbb{R}^3$ of $i$ -th proximal anchor point of the machine frame
$\mathbf{b}_i$	Position vector $\in \mathbb{R}^3$ of $i$ -th distal anchor point of the mobile platform
$\beta_R$	Wrapping angle of the cable on the pulley
$\mathbf{c}$	Vector of calculation variables in the constraint satisfaction problem
$\mathbf{C}$	Compliance matrix $\in \mathbb{R}^{6 \times 6}$ of the cable robot in operational space
$\mathcal{C}$	Set $\subset \mathbb{R}^m$ of the $m$ -dimensional hypercube of feasible forces in the cables
$\gamma_R$	Rotation of the panning pulley about the $z$ -axis
$d_D$	Diameter of the drum
$E_C$	Young's modulus of the cable
$\mathbf{f}$	vector $\in \mathbb{R}^m$ collecting all cable forces as generalized forces
$\mathcal{F}$	Set $\subset \mathbb{R}^m$ with feasible solutions for the force distribution problem
$f_{\min}$	Required pretension in the cable
$f_{\max}$	Feasible maximum tension in the cable
$f_H$	Horizontal part of the cable force $\mathbf{f}_i$ when considering cable sagging
$\mathbf{f}_P$	Force applied to the platform
$\varphi^{\text{IK}}$	$\mathbb{R}^n \rightarrow \mathbb{R}^m$ the mapping defining the inverse kinematics transformation
$\varphi^{\text{DK}}$	$\mathbb{R}^m \rightarrow \mathbb{R}^n$ the mapping defining the forward kinematics transformation
$\Phi^C$	$\mathbb{R}^2 \rightarrow \mathbb{R}^2$ the mapping of the sagging cable model
$\Phi^G$	$\mathbb{R}^{n_D} \rightarrow \mathbb{R}^{6m}$ mapping for parameterization for robot geometry
$\Phi$	Mapping of inequality constraints in the constraint satisfaction problem
$g$	Gravity acceleration
$g_C$	Specific gravity force of the cable per length
$\mathbf{g}^C$	Vector $\in \mathbb{R}^n$ of generalized centripetal and Coriolis forces
$h_D$	Pitch of the groove of the drum
$\mathbf{H}$	Matrix $\in \mathbb{R}^{m \times r}$ with a spanning base of the kernel of $\mathbf{A}^T$
$\mathbf{h}_i$	Spanning base vector $\in \mathbb{R}^m$ of the kernel $\mathbf{H}$ of $\mathbf{A}^T$

<b>I</b>	Square identify matrix
$I_D$	Moment of inertia of the drum
$I_M$	Moment of inertia of the servo motor
$I_P$	Inertia tensor $\in \mathbb{R}^{3 \times 3}$ of the mobile platform
$I_{PG}$	Moment of inertia of the planetary gearbox
$I_R$	Moment of inertia of the pulley
$I_S$	Moment of inertia of the spooling unit
$I_W$	Effective overall moment of inertia of the winch
$i_{dq}$	Current of the servo motor
<b>J</b>	Kinematic Jacobian $\in \mathbb{R}^{n \times m}$ of the robot
$J_A$	Jacobian of the closure constraints $v$ w.r.t. changes in the pose
$J_B$	Jacobian of the closure constraints $v$ w.r.t. changes in the cable length
$J_v$	Jacobian of forward kinematics constraints $v$ w.r.t. changes in the pose
<b>K<sub>C</sub></b>	Stiffness matrix $\in \mathbb{R}^{m \times m}$ of the robot in configuration space
<b>K<sub>G</sub></b>	Geometric stiffness matrix $\in \mathbb{R}^{6 \times 6}$ of the robot in operational space
<b>K<sub>O</sub></b>	Linear elastic stiffness matrix $\in \mathbb{R}^{6 \times 6}$ of the robot in operational space
<b>K<sub>OS</sub></b>	Stiffness matrix $\in \mathbb{R}^{6 \times 6}$ of the robot in operational space
$\mathcal{K}_0$	Fixed world coordinate system
$\mathcal{K}_i$	Coordinate system associated with point $i$
$\mathcal{K}_P$	Reference coordinate system attached to the mobile platform
$\mathcal{K}_{A,i}$	Local coordinate system of the $i$ -th proximal anchor point $A_i$
$\mathcal{K}_{B,i}$	Local coordinate system of the $i$ -th distal anchor point $B_i$
$k_i$	Stiffness coefficient of the $i$ -th cable
$k'_C$	Specific stiffness of the cable
$k_A$	Stiffness of the winch drivetrain
$\mathbf{l}_i$	Vector $\in \mathbb{R}^3$ of the $i$ -th cable
$l_i$	Length of the $i$ -th cable
<b>l</b>	Vector $\in \mathbb{R}^m$ collecting all cable lengths as generalized coordinates
<b>L</b>	Diagonal matrix $\in \mathbb{R}^{m \times m}$ with the cable lengths as elements
$L$	Lagrangian function
<b>L<sub>12</sub></b>	Winding induction of the servo motor
$m$	Number of cables of the robot
<b>m</b>	Vector $\in \mathbb{R}^3$ of the center of the workspace used for hull projection
$m_G$	Mass of linear traveling carriage (spooling unit) of the winch
$m_P$	Mass of the mobile platform
$n$	Degrees-of-freedom of the mobile platform
$n_R$	Number of pulleys in the drivetrain
$n_W$	Number of windings on the drum
$n_D$	Number of design parameters
$v$	$\mathbb{R}^m \rightarrow \mathbb{R}^n$ mapping defining the robot's kinematic closure constraints
$v_C$	Effective density of a cable cross section
$v_{PG}$	Gear ratio of the winch's planetary gearbox
$v_W$	Overall transmission ratio of the winch
<b>P</b>	Transformation matrix $\in \mathbb{R}^{3 \times 4}$ for quaternion in dynamics

$\mathcal{P}$	Set of measurement poses for calibration
$\psi_{dq}$	Flux linkage of the servo motor
$\mathcal{Q}$	Set $\subset \mathbb{R}^n$ of wrenches to be generated by the robot
$\mathbf{Q}$	Quaternion for parameterization of $SO_3$
$\mathbf{q}$	Vector of the generalized coordinates in dynamics
$r$	Degree-of-redundancy of a robot
$\mathbf{r}$	Position of the platform where $\mathbf{r} \in \mathbb{R}^3$
$\mathbf{r}_M$	Mobile platform's center of gravity $\in \mathbb{R}^3$
$r_C$	Radius of the cable
$r_D$	Radius of the drum of the winch
$r_R$	Radius of the pulley
$\mathbf{R}$	Orientation of the platform where $\mathbf{R} \in SO_3$
$\mathcal{R}_0$	Set $\subset SO_3$ of orientation matrices
$R_{12}$	Ohmic resistance of the servo motor windings
$\rho_C$	Density of the cable
$\rho'_C$	Linear density of the cable per length
$s$	Sagging of the cable
$\mathcal{S}$	Set $\subset \mathbb{R}^m$ of the solution space of feasible force distributions
$SO_3$	Set of the special orthogonal group with all spatial rotations
$SE_3$	Euclidian displacement group with all spatial rigid body transformations
$T$	Kinematic energy of the cable robot
$T_P$	Kinematic energy of the mobile platform
$T_W$	Torque in the winch
$T_M$	Servo motor torque
$T_F$	Friction torque in the winch
$\mathbf{T}$	Transformation matrix $\in \mathbb{R}^{6 \times 6}$
$\theta_{eff,i}$	Shaft/drum angle of the $i$ -th motor/winch
$\theta_D$	Rotation angle of the winch drum
$\Theta_D$	Vector $\in \mathbb{R}^m$ collecting the rotation angles of all winch drums
$\theta_M$	Rotation angle of the servo motor
$\boldsymbol{\tau}_P$	Torque vector $\in \mathbb{R}^3$ applied to the mobile platform
$U$	Potential energy of the robot
$U_P$	Potential energy of the mobile platform
$U_i$	Potential energy of the $i$ -th cable
$\mathbf{u}_i$	Unit vector $\in \mathbb{R}^3$ of the $i$ -th cable
$\hat{\mathbf{u}}_i$	Unit vector $\in \mathbb{R}^3$ of the effective direction of $i$ -th cable under sagging
$\mathbf{u}_{dq}$	Input voltage of the servo motor
$\mathbf{v}$	Vector of verification variables in the constraint satisfaction problem
$\mathbf{w}_P$	Vector $\in \mathbb{R}^n$ of the applied wrench of the platform
$\mathcal{W}$	Set of poses in the respective motion group representing the workspace
$\mathcal{X}_c$	Solution set of the constraint satisfaction problem
$\mathcal{X}_s$	Search space is a set used with constraint satisfaction problem

$\mathcal{X}_v$	Verification domain is a set for the constraint satisfaction problem
$\mathbf{y}$	Pose as generalized coordinate vector $\in \mathbb{R}^n$ of the platform
$Z_P$	Pole pair number of the servo motor

## Subscripts

A	Proximal anchor point on the machine frame
B	Distal anchor point on the mobile platform
C	Cable
D	Drum
M	Motor
P	Platform
R	Pulley (roller), not to be confused with platform
W	Winch
DK	Direct kinematics/forward kinematics
IK	Inverse kinematics
FD	Forward dynamics
ID	Inverse dynamics
OS	Operational space
CO	Constant orientation (workspace)
TO	Total orientation (workspace)