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Pareto-Nash-Stackelberg Game and Control Theory

Intelligent Paradigms and Applications

Springer

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To

Valentina, Dorin, Mihai and Zina
Mihail, and Vladimir

Preface

This monograph is a result of prolonged work in the domain of game theory for nearly twenty years. It is dedicated to non-cooperative or strategic form games [1], simultaneous and sequential games [2], their mixtures and control [3]. Considered models are appropriate to games in extensive form [4–7]. Nevertheless, main interests remain essentially in the area of strategic form games.

The mathematical background for the book is somewhat more advanced than that for postgraduate students of the applied mathematics departments. It needs basic knowledge and skills from game theory, optimization methods, multi-criteria optimization, optimal control theory and fundamental mathematical disciplines as linear algebra, geometry, calculus and probability theory. Additionally, the book needs some knowledge of computer science foundations and the Wolfram language.

It must be mentioned that selected topics from all three parts of the book have been taught as an advanced specialization course "*Socio-Economical Problems and Game Theory*" for master's degree students of the applied mathematics specialization at Faculty of Mathematics and Computer Science at Moldova State University. Selected topics from this book were taught in the "*Game Theory*" and "*Operational Research*" courses for students of the same faculty.

The monograph consists of fifteen chapters divided into three parts that are dedicated respectively to non-cooperative games, mixtures of simultaneous and sequential multi-objective games, and to multi-agent control of Pareto-Nash-Stackelberg type. The Introduction chapter presents an overview. The book contains also the Bibliography, an Index, and a List of Symbols.

The book title may be seen as a compromise taken with the aim to have a short monograph's name which will reflect simply its content. It is an approximate synonym for the longer names "*theory of multi-objective multi-agent simultaneous and sequential games and optimal control mixtures*" or/and "*theory of multi-objective multi-leader multi-follower games and multi-agent control*". Sure, such names are seemed to be less acceptable. So, monograph's title was selected in order to be short and clear by associating it with the names of personalities who initiated well known branches of mathematics:

- **Pareto**—multi-objective/multi-criteria optimization,
- **Nash**—strategic/normal form simultaneous games,
- **Stackelberg**—strategic/normal form sequential games,
- **control of Pareto-Nash-Stackelberg type**—multi-objective multi-agent control taken as a mixture of simultaneous and sequential decision process in order to control states of a system.

The formal language used to expose *Pareto-Nash-Stackelberg game and control theory* is generally common for the enumerated above domains of mathematics. Nevertheless, its distinct features consist of being at the same time descriptive, constructive and normative [8]. More the more, the theory has its distinct and specific topics, models, concepts, problems, methods, results and large areas of investigations, extensions, and implementations. The purposes of the present work consist mainly and essentially of highlighting the mathematical aspects of the theory.

The monograph was prepared by the author himself in LaTeX. He is really conscious that it may admit some imperfections. So, suggestions, comments and observations are welcomed in order to continue efficiently investigations in a large spectrum of theoretical and practical unsolved problems. Undoubtedly, they will be treated very seriously, with great care and gratitude.

The book is addressed to researchers and advanced students both in mathematical and applied game theory, as well as multi-agent optimal control. Exposed theoretical results may have direct implementations in economic theory and different areas of human activity where strategic behaviour is underlying.

Chișinău, Moldova Valeriu Ungureanu
November 2017

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Contents

Symbols and Abbreviations

$\mathbb{N}$	Set of natural numbers
$\mathbb{N}^*$	$\mathbb{N}^* = \mathbb{N}\backslash\{0\}$
$\mathbb{N}^n$	Cartesian product of n sets $\mathbb{N}$
$\mathbb{Z}$	Set of integer numbers
$\mathbb{Q}$	Set of rational numbers
$\mathbb{R}$	Set of real numbers; real line
$\alpha, \beta, \ldots$	Notation for real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
N	$N = \{1, 2, \ldots, n\}$ is a set of players $1, 2, \ldots, n$
A^T	The transpose of a matrix A
$\mathbf{x} = x$	$\mathbf{x} = x = (x_1, x_2, \ldots, x_n)^T$— notation for $x \in \mathbb{R}^n$
$x \geq 0$	Sign constraints on x
$x > 0$	The components of vector x are positive
$x \geq y$	At least one component of x is strictly greater than the correspondent component of y
$x \geqq y$	Components of x are greater than components of y or they may be all equal
$x_i \gtreqless 0$	Greater than 0, equal to 0 or less than 0
$\alpha \gg \beta$	α is much greater than β
X, Y	Notation for subset of $\mathbb{R}^n$
$X \subseteq Y$	X is a subset of Y
$X \subset Y$	X is a proper subset of Y
$x \in X$	x is an element of X
$X \cap Y$	Intersection of X and Y
$X \cup Y$	Union of X and Y
$\|X\|, \#X$	Cardinality (power) of X
$\dim(X)$	Dimension of X
$f : X \to R$	Function f
$f(x),\ x \in X$	Alternative function notation
$f : X \multimap Y$	Multivalued mapping f

$\mathscr{P}(X)$	Power set of X
$\mathrm{gr} f(x)$	Graph of $f(x)$
$\mathrm{epi} f(x)$	Epigraph of $f(x)$
$\mathrm{hyp} f(x)$	Hypo-graph of $f(x)$
Γ	Normal form game $\Gamma = \langle N, \{X_p\}_{p \in N}, \{f_p(x)\}_{p \in N} \rangle$
Gr_p	Graph of *pth* player best response mapping
$NES(A, B)$	Nash Equilibrium Set function of A and B
γ^A, γ^B	Knowledge vectors
$\Gamma_{\gamma^A \gamma^B}$	Game with knowledge vectors γ^A *and* γ^B
$\hat{x}^{-p}$	$\hat{x}^{-p} = (\hat{x}^1, \hat{x}^2, \ldots, \hat{x}^{p-1}, \hat{x}^{p+1}, \ldots, \hat{x}^n)$
X_{-p}	$X_{-p} = X_1 \times X_2 \times \ldots \times X_{p-1} \times X_{p+1} \times \ldots \times X_n$
$\hat{x}^p \Vert \hat{x}^{-p}$	$\hat{x}^p \Vert \hat{x}^{-p} = (\hat{x}^p, \hat{x}^{-p}) = (\hat{x}^1, \hat{x}^2, \ldots, \hat{x}^{p-1}, \hat{x}^p, \hat{x}^{p+1}, \ldots, \hat{x}^n) = \hat{x}$
$f'(x)$	Gradient of $f(x)$
$O(\alpha)$	Infinitesimal of higher order than α
$L(x; u_0, u)$	Lagrangian function
$x^T y$	Scalar product of x and y
$\langle x, y \rangle$	Scalar product of x and y
$A_{m \times n}$	Matrix $A[m \times n]$ with m rows and n columns
a_{ij}	Element of a matrix A
AB	Matrix product
A^{-1}	Inverse of A
E	Identity matrix
$\mathrm{rank}(A)$	Rank of A
$\det(A)$, $\|A\|$	Determinant of A
$\Vert x \Vert$	$\Vert x \Vert = \sqrt{x_1^2 + \ldots + x_n^2} = \sqrt{x^T x}$ is the Euclidean norm of $x \in \mathbb{R}^n$
$\rho(x, y)$	$\rho(x, y) = \Vert x - y \Vert$ is the distance between x and y
$V_\varepsilon(x^*)$	$V_\varepsilon(x^*) = \{x \in X \mid \Vert x - x^* \Vert < \varepsilon\}$ is the ε-neighbourhood of x^*
$\mathrm{int}(X)$	Interior of a set X
$\bar{X}$	Closer of a set X
$\lfloor x \rfloor$	Integer part of $x \in R$; Maximal integer not greater than $x \in R$
$\lceil x \rceil$	Minimal integer not smaller than $x \in R$
$\{x\} = x - \lfloor x \rfloor$	Fractional part of $x \in R$
$\exists x$	Existential quantifier: “exists at least one” x
$\forall x$	Universal quantifier: “for all” x
$\mid$, :	With properties
$\square$	End of a proof
$:=$, $=$	Assignment operators
$1^\circ, 2^\circ, \ldots$	Numbering steps of an algorithm
MNT	Maximin-Nash taxon
MT	Maximin taxon
NES	Nash Equilibrium Set
NSE	Set of Nash-Stackelberg equilibria
NT	Nash taxon
ONT	Optimum-Nash taxon

OST	Optimum-Stackelberg taxon
OT	Optimum taxon
PE	Set of pseudo-equilibria
PNES	Pareto-Nash equilibrium set
PPAD	Polynomial Parity Argument for Directed graphs
SNSE	Set of safe Nash-Stackelberg equilibria
SSES	Safe Stackelberg equilibrium set
ST	Stackelberg taxon
TSP	Traveling salesman problem
TSPT	Traveling salesman problem with transportation and fixed additional payments
USES	Unsafe Stackelberg equilibrium set