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Thomas Wriedt · Yuri Eremin
Editors

The Generalized Multipole Technique for Light Scattering

Recent Developments

 Springer

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Preface

Computational electromagnetics is a very rapidly developing field that developed many theoretical approaches and computational tools. Over the years, it has extended its range of application from microwave, light scattering to nanophotonics and even electron energy loss spectroscopy. The Generalized Multipole Technique (GMT) is a surface-based theory which is not that well known but there are a couple of researchers continuously developing and extending the method such that it has reached some kind of maturity over the years.

In 1998, we arranged a workshop [1] in Bremen, Germany supported by the Volkswagen Foundation which had a focus on the Generalized Multipole Technique to document the state of the method at that time and to especially initiate discussion between the different research groups. Following the workshop, an edited volume [2] was published with contributions by major researchers in the field. Out of this workshop, a number of international collaborations arose which continued to work on different variants of the Generalized Multipole method.

The name Generalized Multipole Technique (GMT) was coined by Art Ludwig [3] for a number of related methods to solve the electromagnetic boundary value problem, which were developed independently by a number of research groups distributed all over the world. The common feature of these methods consists in field expansion by a number of multipoles positioned away from the boundary surface. Commonly, some kind of generalized point matching scheme is applied to find the expansion coefficient of the multipoles.

Over the years, research in the GMT continued and many new advances in theory, programming, and application have been achieved such that after 20 years, we think it is the right time to have another close look at the current state of the method. The edited book compiles a couple of chapters on various concepts related to the General Multipole Technique to demonstrate the progress achieved over the last two decades and show the new ideas developed during the last 10 years.

In Chap. 1, it is shown that the theory of principal modes can be derived for any smooth particle starting from a set of distributed electric and magnetic multipoles.

An important field of development is hybrid methods. That the invariant imbedding approach can be combined with the null-field method is demonstrated in Chap. 2.

The Null-field Method with Discrete Sources (NFM-DS) makes use of field expansion using multiple multipoles. Recent progress to compute light scattering by large axisymmetric particles using NFM-DS is presented in Chap. 3.

Some practical applications in nanotechnology require light scattering simulation by a particle partly embedded in an infinite stratified medium. How this problem can be handled using the Discrete Sources Method is investigated in Chap. 4.

Chapter 5 is an overview of the recent works in the Method of Auxiliary Sources.

Chapter 6 presents a novel numerical approach to investigate the resonance behavior of plasmonic particles on a substrate under electron beam illumination based on the Multiple Multipole Program.

Low-Loss Electron Energy Loss Spectroscopy is currently a hot research topic. How the Generalized Multipole Technique (GMT) can be used in this field is addressed in Chap. 7.

Yasuuras Method of Modal Expansion has been developed in Japan. In Chap. 8, this method is applied to investigate scattering by gratings.

An important aspect of the Generalized Multipole Technique is the suitable choice of locations for the sources. This topic is treated in Chap. 9 by James E. Richie.

We hope that these chapters give a fresh look at the evolution and development of the Generalized Multiple Technique. Of course in such a book, the fundamentals needed cannot be fully covered. For this, the interested reader is referred to the book by Doicu et al. [4].

As no book can be published without some assistance, we have to thank all contributors who send their text on time. We especially like to thank Prabhan Vishwanath who helped with latex compilation.

Bremen, Germany
Moscow, Russia

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Acronyms

Al	Aluminum
Al ₂ O ₃	Alumina
AOI	Angle of Incidence
AS	Auxiliary Surface
Au	Gold
BEM	Boundary Element Method
CHIEF	Combined Helmholtz Integral Equation Formulation
CMASIE	Continuous Method of Auxiliary Integral Equation
CYM	Conventional Yasuuras Method
DDA	Discrete Dipole Approximation
DFT	Discrete Fourier Transform
DGTD	Discontinuous Galerkin Time Domain
DSCS	Differential Scattering Cross Section
DSIEM	Dual-Surface Integral Equations Method
EBW	Effective Spatial Bandwidth
EBC	Extended Boundary Condition
EBCM	Extended Boundary Condition Method
EELS	Electron Energy Loss Spectroscopy
EIE	Extended Integral Equation
FCM	Filamentary Current Method
FDTD	Finite Difference Time Domain
FEM	Finite Element Method
GLMT	Generalized Lorenz–Mie Theory
GMT	Generalized Multipole Technique
GPMT	Generalized Point Matching Technique
GPU	Graphics Processing Unit
GT	Green Tensor
LHS	Left-Hand Side
MAS	Method of Auxiliary Sources
MFS	Method of Fictitious Sources

MMP	Multiple Multipole Program
MPI	Message Passing Interface
NFM	Null-Field Method
NFM-DS	Null-Field Method with Discrete Sources
PEC	Perfect Electric Conductor
SFS	Scattered Field Singularity
SIE	Surface Integral Equation
SNOM	Scanning Near-Field Optical Microscopy
STEM	Scanning Transmission Electron Microscope
SVD	Singular Value Decomposition
SVWF	Spherical Vector Wave Function
TBB	Threading Building Blocks
TE	Transverse Electric
TM	Transverse Magnetic
VIE	Volume Integral Equations