

Complements of Higher Mathematics

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Preface

It is our belief that this book will help many students and researchers with mastering applied problems from different fields. The material presented here stems from courses and seminars on special topics in mathematics given by the authors for many years at universities, especially at technical, engineering, and economics faculties. We limit our considerations in this chapter to the basic notions and results of each domain of mathematics considered in this volume due to the diversity of the included chapters. Chapter 1 is devoted to complex functions. Here, emphasis is placed on the theory of holomorphic functions, which facilitate understanding the role played by the theory of complex variables in mathematical physics, especially in the modeling of plane problems. Furthermore, we consider the remarkable importance that the theories of special functions, operational calculus, and variational calculus have. A great part of the book is dedicated to second-order partial differential equations, since they are widely used to model phenomena in physics and engineering. In the last chapter, we discuss the basic elements of one of the most modern areas of mathematics, namely the theory of optimal control. To achieve a relative independence of the book, each chapter introduces the necessary mathematical background, i.e., topics from mathematical analysis, topology, functional analysis, and so on, which are used in the other chapters. For this reason, the larger part of this book is accessible to students of technical, engineering, and economics faculties, and researchers working in these fields. Some applications are included to illustrate the theory discussed. The methods used in the book permit the analysis of both theoretical and practical cases, thus offering results of interest to the students of technical, engineering, and economics faculties and, also, for young researchers, interested of higher mathematics.

Our intention was to help the reader to proceed more easily to the study of special topics in mathematics, which is usually studied in the second year of all technical faculties. A number of supplementary topics included in this book have been chosen particularly in consideration of their use in specialized courses.

For the study of this book, it is sufficient for the reader to be familiar with a classical course on mathematical analysis and a standard course on differential geometry and algebra, which usually are included in the first year of most programs.

The authors are aware that there are many more results and even more recent data regarding the domains which are not presented herein.

Only their simple enumeration, even in a simplified form to become more accessible, would have considerably enlarged the present book.

Excluding these allowed the authors to present thorough mathematical proofs of the results presented in the book.

The authors would be grateful for readers' comments on the content and the design of the textbook. We would also be pleased to receive any other suggestions the readers may wish to make.

We express our profound gratitude to Prof. C. Marinescu of the Department of Mathematics, Transilvania University of Brasov, for his kindness in reading the manuscript and making pertinent observations, which were taken into consideration.

We are grateful also for suggesting to write this book with the purpose of supplying the bibliographical material for students interested in higher mathematics.

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Esslingen am Neckar, Germany
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