Advances in Delays and Dynamics

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Delay systems are largely encountered in modeling propagation and transportation phenomena, population dynamics and representing interactions between interconnected dynamics through material, energy and communication flows. Thought as an open library on delays and dynamics, this series is devoted to publish basic and advanced textbooks, explorative research monographs as well as proceedings volumes focusing on delays from modeling to analysis, optimization, control with a particular emphasis on applications spanning biology, ecology, economy and engineering. Topics covering interactions between delays and modeling (from engineering to biology and economic sciences), control strategies (including also control structure and robustness issues), optimization and computation (including also numerical approaches and related algorithms) by creating links and bridges between fields and areas in a delay setting are particularly encouraged.

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Limits of Stability and Stabilization of Time-Delay Systems

A Small-Gain Approach

Springer

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To my parents and husband

—Jing Zhu

To my parents and children

—Tian Qi

To my parents

—Dan Ma

To my parents

—Jie Chen

Preface

Time delays are a prevailing scene in natural and engineered systems. Modern interconnected networks are especially prone to long and variable delays; systems and networks in this category are many, ranging from communication networks, sensor networks, multi-agent systems, cyber-physical systems, biological systems, to networked control systems, to name a few. Except in rare instances, time delays are likely to result in degraded performance, poor robustness, and even instability, which consequently pose significant challenges to the analysis and design of control systems under delayed feedback.

While a recurring subject of study, over the last two decades or so there have been particularly notable advances in the stability analysis of time-delay systems, thanks to the development of analysis methods drawing upon robust control theory and the development of computational methods in solving *linear matrix inequality* (LMI) problems. An extraordinary volume of the literature is in existence on stability problems, and various time- and frequency-domain stability criteria have been developed. Of these developments, while an overwhelming majority of the available results are obtained based upon time-domain Lyapunov–Krasovskii methods and require the solution of LMIs, frequency-domain conditions in the spirit of small-gain theorem have also been sought after. Generally, time-domain stability conditions are applicable to both constant and time-varying delays, but are known to suffer from a varying degree of conservatism. In contrast, frequency-domain tests are largely restricted to constant delays though they often provide tight conditions and appear more susceptible to feedback synthesis.

Despite the considerable advances in stability analysis, control design problems for time-delay systems prove far more challenging. Feedback stabilization of time-delay systems poses a difficult problem and has been somewhat an underdeveloped research area. Fundamental robustness issues have been seldom investigated as well. Furthermore, recent advances in broad fields of science and engineering brought forth new issues and problems to the area of time-delay systems; time delays resulted from the interconnected systems and networks present new challenges unexplored in the past and are increasingly seen to have far more grave effects, which the existing theories do not seem to be well equipped with.

Among other challenges, these issues have led to our work that forms the core of the present book.

We present in this monograph a study on fundamental limits and robustness of stability and stabilization of time-delay systems, with an emphasis on time-varying delay, robust stabilization, and newly emerged areas such as networked control and multi-agent systems. We develop systematically an operator-theoretic approach that departs from both the traditional algebraic and the currently pervasive LMI solution methods. This approach is built on the classical small-gain theorem and is of a distinctive flavor of robust control, which enables us to draw upon rich tools and techniques from robust control theory. The book is organized as follows:

In Chap. 1, we provide a number of motivating examples of both classical and contemporary interest, together with the concise literature survey most relevant to the book contents. Chapter 2 collects some key mathematical facts and results required in the subsequent developments, including the small-gain theorem and rudiments of robust optimal control. In Chap. 3, we develop stability conditions for linear systems subject to time-varying delays. Leveraging on the small-gain theorem, we cast the stability problem as one of robust stability analysis and derive accordingly $\mathcal{L}_2$- and $\mathcal{L}_\infty$-type stability conditions reminiscent of robust stability bounds typically found in robust control theory. The development shows that a variety of stability conditions, both existing and new, can be unified in the form of scaled small-gain conditions, which, other than their conceptual appeal, can be checked using standard robust control toolboxes.

Chapter 4 studies stabilization problems for linear systems subject to unknown variable delays. We investigate the fundamental limit of stabilization by linear time-invariant (LTI) controllers. This problem, commonly referred to as the *delay margin* problem, concerns the limitation for a LTI controller to robustly stabilize a time-delay system, addressing the question: What is the largest range of delay such that there exists a feedback controller capable of stabilizing all the plants for delays within that range? Chapter 4 focuses on single-input single-output (SISO) systems with a constant unknown delay. Drawing upon analytic interpolation theory and rational approximation techniques, we develop fundamental limits on the delay margin. The results are subsequently extended to systems with time-varying delays in Chap. 5, which display a significantly increased level of intricacy and complexity. Chapter 6 focuses on the delay margin achievable by PID controllers, where by common practice, we examine low-order systems; PID controllers are favored for their ease of implementation and are widely used in controlling industrial processes. In Chap. 7 we generalize the delay margin to the notion of *delay radius*, which concerns the range of nonuniform, multivariate delays for multi-input multi-output (MIMO) systems. Bounds and estimates are obtained in a cohesive, unified manner, which in most cases amount to solving an eigenvalue problem.

Chapters 8 and 9 then progress to contemporary topics on networked control and multi-agent systems. Specifically, Chap. 8 studies networked feedback stabilization problems over lossy communication links, where time delay may result from the system itself or from the communication channel. Networked control is broadly referred to as such a mechanism in which control tasks are executed by exchanging information among system components via some form of communication links. Since information transmission cannot be ideal and is in general noisy, communication losses arise. We model the lossy communication channels by stochastic multiplicative uncertainties, which furnish a general description for such communication losses such as data dropout and fading. The problem under consideration is to determine the fundamental threshold of communication noises and uncertainties, so that the delay system can be stabilized robustly. Based on the mean-square small-gain theorem, we derive necessary and sufficient conditions for a system to be stabilizable under a mean-square criterion, for both SISO and MIMO delay systems.

Chapter 9 studies consensus robustness problems for continuous-time multi-agent systems. In a MAS consensus task, a set of agents coordinate to reach a global common state based on the exchange of local information through a communication network. We assume that the agents in the MAS received certain delayed signals, whereas the delay may arise sheer because of the agent dynamics or due to communication delay between the agents. The central issue under our study is the effect of the delayed information exchange on consensus: Can consensus still be achieved under delayed feedback of an agent's neighbor information? To maintain consensus, how will delay constrain network topology? For a given topology, what is the largest possible range of delay allowed in order to insure robustness of consensus? Drawing upon concepts and techniques from robust control, notably those concerning gain margin and gain–phase margin optimizations, we derive robustness conditions for general linear agents to achieve consensus under delay effects. The results show that delayed communication between agents will generally hinder consensus and impose restrictions on the network topology.

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Contents

Symbols and Abbreviations

Sets and Spaces

$\mathbb{C}$	The space of complex numbers
$\mathbb{C}_-$	The open left-half of the complex plane
$\mathbb{C}_+$	The open right-half of the complex plane
$\bar{\mathbb{C}}_-$	The closed left-half of the complex plane
$\bar{\mathbb{C}}_+$	The closed right-half of the complex plane
$\mathbb{C}^n$	The space of n-dimensional complex vectors
$\mathbb{C}^{m\times n}$	The space of m by n complex vectors
$\mathbb{R}$	The space of real numbers
$\mathbb{R}^n$	The space of n-dimensional real vectors
$\mathbb{R}^{m\times n}$	The space of n by n real matrices
$\mathbb{R}^n_+$	The n-dimensional space of positive real numbers
$\mathcal{L}_p$	The space of the measurable functions $f : \mathbb{R} \to \mathbb{R}^n$ with $\|f\|_{\mathcal{L}_p} < \infty$
$\mathcal{H}_2$	The subspace of $\mathcal{L}_2$ analytic in $Re(s) > 0$
$\mathcal{H}_2^\perp$	The subspace of $\mathcal{L}_2$ which consists of functions analytic in $Re(s) < 0$
$\mathcal{H}_\infty$	The subspace of $\mathcal{L}_\infty$ analytic in $Re(s) > 0$
$\mathcal{RH}_\infty$	The real rational subspace of $\mathcal{H}_\infty$
supp(f)	The support of a real-valued function $f : \mathbb{X} \to \mathbb{R}$

Vectors and Matrices

x_i	The ith element of vector x
a_{ij}	The unit of matrix A at the intersection of ith row and jth column
I	The identity matrix
D_f	diag$(f_1, \ldots, f_n)$ of an n-tuple of scalars, vectors, and matrices $\{f_1, \ldots, f_n\}$
$\bar{z}$	The conjugate of a complex number z
x^H	The conjugate transpose of a complex vector x
x^T	The transpose of a complex vector x

$\|x\|_p$	The Hölder p-norm of a vector x with $x \in \mathbb{C}^n$
$\|f\|_{\mathcal{L}_p}$	The $\mathcal{L}_p$ norm of a function f with $f \in \mathcal{L}_p$
A^H	The conjugate of a complex matrix A
$Tr(A)$	The trace of matrix A
$\mathcal{F}_l(A,B)$	The lower linear fractional transformation of matrix A and B
$\bar{\sigma}(A)$	The largest singular value of matrix A
$\rho(A)$	The spectral radius of matrix A
$\lambda_{\max}(A)$	The largest real eigenvalue of matrix A
$\bar{\lambda}(M)$	The largest eigenvalue of the Hermitian matrix M
$\angle(u,v)$	The principal angle between the unitary vectors $u, v \in \mathbb{C}^n$
$cos\angle(u,v)$	The value of $\|u^H v\|$
$A \otimes B$	The Kronecker product of matrix A and B. If $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{p\times q}$,

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}$$

Abbreviations

AQM	Active queue management
ARE	Algebraic Riccati equation
FDE	Functional differential equation
GEVP	Generalized eigenvalue problem
IQC	Integral quadratic constraint
LFT	Linear fractional transformation
LKF	Lyapunov–Krasovskii functional
LMI	Linear matrix inequality
LTI	Linear time-invariant
MARE	Modified algebraic Riccati equation
MAS	Multi-agent system
MIMO	Multi-input multi-output
NCS	Networked control system
SISO	Single-input single-output
SNR	Signal-to-noise ratio
TCP	Transmission control protocol