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Raimundo Nonato Araújo dos Santos  
Aurélio Menegon Neto · David Mond  
Marcelo J. Saia · Jawad Snoussi  
Editors

# Singularities and Foliations. Geometry, Topology and Applications

BMMS 2/NBMS 3, Salvador, Brazil, 2015

 Springer

*Editors*

Raimundo Nonato Araújo dos Santos  
ICMC  
University of São Paulo  
São Carlos, São Paulo  
Brazil

Marcelo J. Saia  
ICMC  
University of São Paulo  
São Carlos, São Paulo  
Brazil

Aurélio Menegon Neto  
Department of Mathematics  
Federal University of Paraíba  
João Pessoa, Paraíba  
Brazil

Jawad Snoussi  
Instituto de Matemáticas, Unidad  
Cuernavaca  
Universidad Nacional Autónoma de México  
Cuernavaca, Morelos  
Mexico

David Mond  
Mathematics Institute  
University of Warwick  
Coventry  
UK

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# Preface

The last mathematical proceedings of an event held at the Institute of Mathematics of the Federal University of Bahia were the famous volume of the “SALVADOR SYMPOSIUM ON DYNAMICAL SYSTEMS,” organized in 1971 by Jacob Palis, Elon L. Lima, and Mauricio Matos Peixoto and published in 1973.

As is well known, several of the papers published there have played an important role in the areas of Dynamical Systems, Singularity Theory, and Foliation Theory.

Forty-six years on (almost a half-century), it would be wonderful to present some mathematical developments following on from those foundations.

This volume of proceedings comes as an initiative in that direction.

In fact, this book is a result of the joint organization, by the Brazilian and Mexican research groups in Singularities and Foliations, of two international mathematical meetings in the Brazilian Northeast: the 3rd Singularity Theory Meeting of Northeast region (or 3<sup>o</sup> ENSINO), from July 8 to 11, 2015, and the Brazil-Mexico 2nd Meeting on Singularities, from July 13 to 17, 2015.

The choice of the city of Salvador was intended to promote the development of research in Singularities and Foliations in the Northeast of Brazil.

The organization from the Brazilian side was carried out by the Singularities group of ICMC-USP/São Carlos, the groups from the Federal Universities of Paraíba and Ceará, and Simone Moraes, Kleiber Cunha and Evandro Carlos dos Santos from the Institute of Mathematics of the Federal University of Bahia.

From the Mexican side, the organizers were F. Aroca, E. Rosales, J. Seade, and J. Snoussi from Instituto de Matemáticas UNAM, together with X. Gómez-Mont from CIMAT and A. Giles Flores from Universidad Autónoma de Aguascalientes.

The meetings brought some of the leading researchers from Brazil and Mexico and from elsewhere, to the Northeast of Brazil, to discuss the latest developments in Singularity Theory, Foliations, and associated areas.

Since the 1980s, there has been a strong collaboration between Mexican and Brazilian researchers in Singularities and Complex Dynamical Systems. Current interactions involve researchers and students from São Carlos, João Pessoa, and Fortaleza, on the Brazilian side, and from the Institute of Mathematics of Cuernavaca (UNAM), CIMAT, and the Faculty of Science of the UNAM in

Mexico. The 1st Brazil/Mexico meeting on Singularities took place in Querétaro, Mexico, from August 1 to 3, 2013, with the participation of 30 researchers and students from both sides.

During the 3rd Singularity Theory Meeting of Northeast region, a school was offered for young researchers and graduate students at master's and Ph.D. levels, with minicourses delivered by Nicolas Dutertre (Université Aix-Marseille—France), Terence Gaffney (Northeastern University—USA), David Mond (University of Warwick—England), and Juan J. Nuño-Ballesteros (Universitat de Valencia—Spain).

During the Brazil-Mexico 2nd Meeting on Singularities, plenary talks were delivered by participants from Brazil and Mexico, as well as by invited mathematicians from different countries, as described below.

**Brazilian team:** Abramo Hefez (UFF), Alexandre Fernandes (UFC), Alice Libardi (UNESP), Aurélio Menegon (UFPB), Jonny Ardila (UFRJ), Leonardo Câmara (UEFS), Marcelo Escudeiro (UEM), Marcio Soares (UFMG), Marcos Craizer (PUC), Maria Elenice (UEM), M. Salarinoghabi (ICMC), N. Thuy (UNESP), Roberta Wik Atique (ICMC), Rodrigo Mendes (UFC).

**Mexican team:** Agustín Romano (UNAM), Edwin Leon (CONACyT/CIMAT), Ernesto Rosales (UNAM), Federico Sánchez-Bringas (UNAM), Fuensanta Aroca (UNAM), Jessica Jáurez (UNAM), José-Luis Cisneros-Molina (UNAM), Mirna Gómez (UNAM), Daniel Duarte (CONACyT/UAZ), José Seade (UNAM), Jawad Snoussi (UNAM), L. Nunes (CIMAT).

### **Invited Speakers from the International Math Community:**

Alexey Remizov (Russia), Carles Bivià-Ausina (Spain), Maria del Carmen Romero Fuster (Spain), Christopher Eyrál (Poland), Jean-Paul Brasselet (France), Kasuto Takao (Japan), Lê Dũng Tráng (France), Mutsuo Oka (Japan), Osamu Saeki (Japan).

Beside the speakers listed above, there were poster sessions presented by postdoctoral researchers and master's and Ph.D. students from all over Brazil, from Mexico, and from many other countries. Further information on the minicourses, plenary talks, and poster presentation sessions and a list of participants may be found via the link

<http://bramexsing2015.icmc.usp.br/index.php/programme>.

We hope this initiative does not stop here and that in the near future it will motivate many other meetings in Singularity Theory and Foliation Theory, contributing to their development and to the development of mathematics in the Northeast of Brazil.

The Brazil/Mexico 3rd Meeting on Singularities was held in Cuernavaca, Mexico, from August 7 to 11, 2017. Further information may be found in the link <http://www.matcuer.unam.mx/3rdMeetingOnSingularities/>.

São Carlos, Brazil  
João Pessoa, Brazil  
Coventry, UK  
São Carlos, Brazil  
Cuernavaca, Mexico

Raimundo Nonato Araújo dos Santos  
Aurélio Menegon Neto  
David Mond  
Marcelo J. Saia  
Jawad Snoussi

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Many people and institutions contributed to the realization of these meetings. Without their help it would certainly have been impossible.

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3. everyone from the International Math Community (England, France, Japan, Russia, Spain and USA) for their mini-courses and talks.

These contributions were fundamental for the richness of the meetings. Thank you so much!

We also thank:

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6. Ms. Terezinha and her nice staff who provided the delicious juice and cakes for the coffee breaks, giving us the opportunity to experience a piece of Bahian

cuisine. Thanks for representing so well one of the nicest part of Bahia's culture. Your kindness, love and smiles make everything possible.

Last but not least, we thank Prof. Evandro Carlos Ferreira dos Santos, Director of the Institute of Mathematics of UFBA for making available the IM installations during the meetings. Also, Profs. Antonio Andrade do E. Santo and Maria Amelia B. Holenwerger of CETEC/UFRB-Bahia, for believing and working together with us from the beginning all the way up to the realization of the meetings.

Thank You All Very Much !!!



# Contents

<b>Part I Lecture Notes: Geometry, Topology, and Algebraic Aspects of Singularities</b>	
<b>Combinatorial Models in the Topological Classification of Singularities of Mappings</b> . . . . .	3
J. J. Nuño-Ballesteros	
<b>Topology of Real Singularities</b> . . . . .	51
Nicolas Dutertre	
<b>Equisingularity and the Theory of Integral Closure</b> . . . . .	89
Terence Gaffney	
<b>Part II Surveys Papers on Advances in Foliations and Singularity Theory: Topology Geometry and Applications</b>	
<b>A Brief Survey on Singularities of Geodesic Flows in Smooth Signature Changing Metrics on 2-Surfaces</b> . . . . .	135
N. G. Pavlova and A. O. Remizov	
<b>Orbital Formal Rigidity for Germs of Holomorphic and Real Analytic Vector Fields</b> . . . . .	157
Jessica Angélica Jaurez-Rosas	
<b>On Singular Holomorphic Foliations with Projective Transverse Structure</b> . . . . .	181
Bruno Scárdua	
<b>Disentanglements of Corank 2 Map-Germs: Two Examples</b> . . . . .	229
David Mond	
<b>Singular Fibers of Stable Maps of Manifold Pairs and Their Applications</b> . . . . .	259
Osamu Saeki and Takahiro Yamamoto	

<b>A Global View on Generic Geometry</b> . . . . .	295
María del Carmen Romero Fuster	
<b>Equisingularity</b> . . . . .	327
Lê Dũng Tráng	
<b>On the Factorization of the Polar of a Plane Branch</b> . . . . .	347
A. Hefez, M. E. Hernandez and M. F. H. Iglesias	
<b>Local Zeta Functions for Rational Functions and Newton Polyhedra</b> . . . . .	363
Míriam Bocardo–Gaspar and W. A. Zúñiga–Galindo	
<b>Symbolic Powers of Ideals</b> . . . . .	387
Hailong Dao, Alessandro De Stefani, Eloísa Grifo, Craig Huneke and Luis Núñez–Betancourt	
<b>Some Open Problems in Complex Singularities</b> . . . . .	433
José Seade	
<b>Part III Selected Papers in Foliations and Singularity Theory</b>	
<b>A Comprehensive Approach to the Moduli Space of Quasi-homogeneous Singularities</b> . . . . .	459
Leonardo M. Câmara and Bruno Scárdua	
<b>On the Roots of an Extended Lens Equation and an Application</b> . . . . .	489
Mutsuo Oka	
<b>A Lefschetz Coincidence Theorem for Singular Varieties</b> . . . . .	513
J.-P. Brasselet, A. K. M. Libardi, T. F. M. Monis, E. C. Rizzioli and M. J. Saia	
<b>Preservation of Immersed or Injective Properties by Composing Generic Generalized Distance-Squared Mappings</b> . . . . .	537
Shunsuke Ichiki and Takashi Nishimura	
<b>Arc Criterion of Normal Embedding</b> . . . . .	549
Lev Birbrair and Rodrigo Mendes	



Participants of the Second Brazil-Mexico Meeting on Singularities and Third Northeastern Meeting on Singularities. Photo taken at the Federal University of Bahia, Salvador, Brazil, in July 2015. *Source:* Paulo César Soares de Oliveira/ICMC-USP