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Dietmar Gross · Thomas Seelig

# Fracture Mechanics

With an Introduction to Micromechanics

Third Edition

 Springer

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# Preface

This book has evolved from lectures on fracture mechanics and micromechanics which we held for students of engineering and materials science over the years. It is primarily meant as an aid for students learning the foundations of these subjects. At the same time this book may also serve as an introduction into these fields for researchers and practitioners in industry and to provide the theoretical background for solving respective problems.

The book covers the most important areas of fracture mechanics and gives an introduction into micromechanics. Our major concern was the presentation of principal concepts and methods in a clear and sound manner as a basis for a deeper entry into the matter. The presentation mainly focuses on the mechanical description of fracture processes; yet, material specific aspects are also discussed. To keep the text self-contained, continuum mechanical and phenomenological foundations are recapitulated first. They are followed by a brief survey of classical fracture and failure hypotheses. A major part of the book is devoted to linear fracture mechanics and elastic-plastic fracture mechanics. Further chapters deal with creep fracture and dynamic fracture mechanics. An extensive chapter treats foundations of micromechanics and homogenization. Finally, elements of damage mechanics and probabilistic fracture mechanics are presented. Suggestions for further reading are listed at the end of each chapter.

The first two editions were well accepted by the readers making a new edition necessary. We have again used this opportunity to completely revise the book and to incorporate a number of extensions. Among others, we have addressed topics such as the numerical treatment of crack and homogenization problems, cracks in anisotropic materials, periodic microstructures and the nonlocal regularization of damage models. Finally, some further examples have been added.

The authors are indebted to all who have contributed to this book. This particularly includes those from whom we have learned or, as Roda Roda has put it ironically: “Copying from four books yields a fifth profound book”. Special thanks go to Mrs. Dipl.-Ing. H. Herbst who has prepared most of the figures. Finally, the pleasant cooperation with the publisher is gratefully acknowledged.

Darmstadt and Karlsruhe in autumn 2017,

Dietmar Gross  
Thomas Seelig

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# Introduction

*Fracture* in science and technology is understood as the total or partial separation of an originally intact body or structure. The characterization of corresponding mechanical phenomena is the subject of fracture mechanics. From an engineering point of view, in many cases a macroscopic approach is sufficient. But also microscopic aspects became of increasing interest during the past years. For both, the macroscopic and microscopic approach, continuum mechanics has proven to be an effective tool. Using this well-developed instrument, fracture criteria and concepts may be established, which allow a prediction of the fracture behavior.

In general, separation of a body occurs by propagation of one or several cracks through the material. Therefore, fracture mechanics deals to a large extent with the behavior of cracks. In a real structure or material, cracks or other defects of different size which possibly evolve to cracks, are virtually always present. One of the main questions which fracture mechanics shall answer is as follows: under what circumstances does a crack in a body start to propagate and subsequently lead to fracture? Other topics are the conditions for crack nucleation, crack path prediction, or the velocity of a propagating crack.

In continuum mechanics, usually stresses and strains are used to describe the mechanical behavior of a solid. These quantities, likewise very important in fracture mechanics, can not always be directly applied for the characterization of fracture processes. One reason for this is that stresses or strains might become infinitely large at a crack tip. Another one follows from the simple fact that two cracks of different lengths behave differently when loaded by the same external stress. Under increasing load the longer crack will start to propagate at a lower stress than the shorter one. For these reasons additional quantities like *stress intensity factors* or the *energy release rate* have been introduced in fracture mechanics. They are able to characterize the local state at a crack tip and the global behavior of the crack during propagation, respectively.

For the understanding of fracture processes, at least a partial insight into the underlying micromechanisms is useful. For example, from observing the changes in the microstructure it becomes understandable how a material defect may increase until it can be regarded as a microscopic or a macroscopic crack. The relevance of

micromechanisms also explains the important role that material science has played in the past and still plays in the evolution of fracture mechanics. Increasingly, microscopic processes are mechanically modeled nowadays and incorporated into continuum theories. Special fields like damage mechanics or micromechanics have been developed from these efforts and have become important tools in fracture mechanics. In particular, micromechanics offers a theoretical framework for a systematic treatment of defects and their influence on different length scales.

Fracture mechanics may be classified from different points of view. Usually it is divided into *linear elastic fracture mechanics* and *nonlinear fracture mechanics*. The first describes fracture processes by using linear elasticity. Since this is appropriate particularly for brittle fracture, linear fracture mechanics also is understood as *brittle fracture mechanics*. In contrast, nonlinear fracture mechanics characterizes fracture processes which are dominated by inelastic material behavior. Depending on whether the material is elastic-plastic or considerable viscous effects are present, a further partition into *elastic-plastic fracture mechanics* and *creep fracture mechanics* is common practice. Another classification is rather material oriented. Accordingly, fracture mechanics sometimes is divided into fracture mechanics of metals, of polymers, or of composites. If, in contrast to a deterministic approach, probabilistic methods are used to characterize fracture processes, we call this *probabilistic fracture mechanics*.

The roots of fracture mechanics reach back to the beginnings of modern mechanics. Already LEONARDO DA VINCI (1452-1519) studied the tensile and bending strength of bars and beams as well as the fracture (tearing) of metal wires. Among other things, he observed that the tensile strength of wires depends of their length, i.e. on the probability of the presence of critical defects. GALILEO GALILEI (1564-1642) in 1638 reflected about the fracture of beams, which led him to the conclusion that the bending moment is the crucial loading measure. In parallel with the evolution of continuum mechanics in the 19th century, a number of different strength hypotheses had been proposed which partly are still in use as fracture or failure criteria. They directly employ stresses or strains to characterize the loading of the material. Corresponding efforts took place at the beginning of the last century in conjunction with the development of plasticity theory. Though K. WIEGHARDT (1874-1924) within the framework of linear elasticity already in 1908 presented an exact description of the stresses in the vicinity of a crack-tip, his findings did not yet lead to a fracture concept. But only in 1920 the first cornerstone of a fracture theory of cracks was set through A.A. GRIFFITH (1893–1963) by introducing the necessary energy for crack growth in the energy balance and by formulating an energetic fracture concept. A further milestone was the statistical theory of fracture formulated in 1939 by W. WEIBULL (1887-1979). But the actual breakthrough was achieved in 1951 by G.R. IRWIN (1907-1998) who was the first to characterize the state at a crack tip by stress intensity factors. The so-called K-concept of linear fracture mechanics rapidly found entrance into practical applications and is meanwhile firmly established. In the early 60s the first concepts for an elastic-plastic fracture mechanics were proposed and a rapid development set in. First steps towards an integration of damage mechanics and micromechanics into fracture mechanics have

been attempted in the 80s. Despite substantial progress, fracture mechanics is by no means an already completed field but still a subject of intensive research.

The development of fracture mechanics is driven to a large extent by the ambition to prevent failure of technical constructions and components. Therefore, fracture mechanics is used as a design tool in all fields where fracture and an accompanying failure of a component with serious, or in the worst case, catastrophic consequences must be prevented. Typical fields of application can be found in aerospace engineering, microsystems technology, nuclear power techniques, pressure vessel construction, automotive engineering, or in steel and solid construction. Moreover, fracture mechanics is used in many other fields for the solution of problems where separation processes play a dominant role. Some examples are comminution technology, earthquake research, and materials science.