

# Emergence, Complexity and Computation

Volume 27

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Pierre-Yves Louis · Francesca R. Nardi  
Editors

# Probabilistic Cellular Automata

Theory, Applications and Future Perspectives

 Springer

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# Preface

*Probabilistic cellular automata* (PCA) are interacting discrete stochastic dynamical systems used as a modelling tool for a wide range of natural and societal phenomena. On the applied side, PCA constitute an attractive computational framework for high-performance computing, distributed computing and simulations. Indeed, PCA have been put to good use as part of multiscale simulation frameworks for studying natural systems or large interconnected network structures. On the mathematical side, PCA have a rich mathematical theory that leads to a better understanding of the role of randomness and synchronicity in the evolution of large systems.

This book attempts to present a wide panorama of the current status of PCA theory and applications. These various contributions cover important subjects seen from probabilistic, statistical mechanical, computational and dynamical system points of view, and illustrated with applications to computer science and natural sciences. It *gathers contributions* from scientists with different perspectives, expectations, backgrounds and techniques. By bringing together works from international experts, this book intends to disseminate terminologies, common knowledge, tools, references and challenges.

This project started with a workshop organised in June 2013 at EURANDOM<sup>1</sup>, TU Eindhoven. The organising committee wants to thank O. Boxma, F. den Hollander and R. van der Hofstad for scientific support. P. Koorn is warmly acknowledged for organisational and administrative help regarding this meeting. The workshop was very well attended by international participants. Presentations produced lively discussions both at theoretical and at applied levels. Feedback was very positive about the event. The opportunity to publish associated papers in a scientific journal was considered. Nevertheless, the strong interdisciplinary aspects were not compatible with the specific scope of a journal. Motivated by the nice atmosphere and proven interest, the organising committee enthusiastically became the editorial board of a *contributed book*. This choice granted more freedom and allowed a more

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<sup>1</sup><http://www.eurandom.nl/events/workshops/2013/PCA/PCA.html>

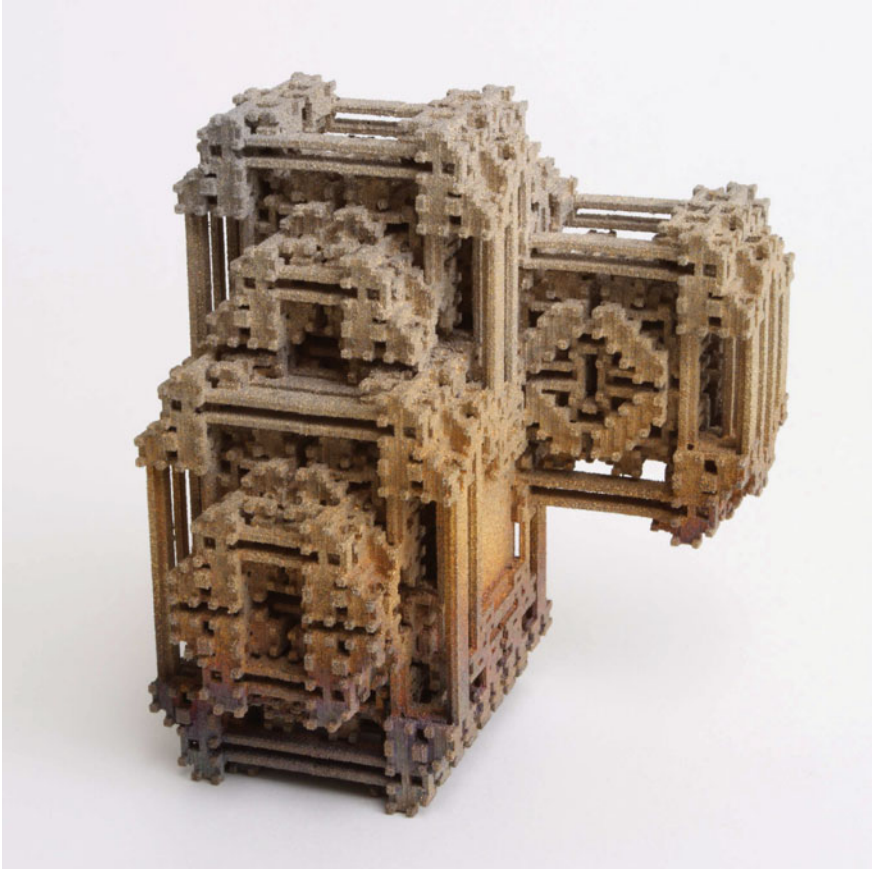
introductory style of the contributions, requiring them to be understandable and useful for scientists in different communities. The board worked on a collective basis sharing the different tasks and applying the respective areas of expertise to develop the three parts. An effort was made to select contributors from different academic status to foster interaction among generations. We are grateful to all senior and junior researchers who responded to this effort.

It is a great pleasure for the editorial board to see this project finalised. We want to thank all the colleagues for their contributions that helped concreting our vision. We acknowledge many referees whose help was decisive. Finally, the main editors Pierre-Yves Louis and Francesca R. Nardi are grateful to the other board members E. Cirillo, N. Fatès, Roberto Fernández, R. Merks, W. Ruzsel and C. Spitoni for their competence and involvement in carrying out this project.

This book is aimed to researchers and motivated students who want to gain more insight into this broad topic. Our purpose is to stimulate cross-fertilisation both at theoretical and at applied levels. We think that the introductory level of this book could be a good starting point for non-specialists wishing to enter the field. The book should also be of interest as a source of challenges and open issues. As emphasised, probabilistic cellular automata is a lively topic, and this book does not claim to be either self-contained or exhaustive. Finally, despite our efforts, some errors, omissions or amendments may have escape our attention. We are grateful for feedback, corrections and comments.

Eindhoven, The Netherlands  
July 2016

Pierre-Yves Louis  
Francesca R. Nardi



Example of probabilistic and deterministic cellular automata in the visual arts. See Chap. 2. Breed 1.2 #e365 (3D print), Driessens & Verstappen, 2007, in private collection



# Acknowledgements

The editorial board warmly thanks the authors and referees whose contribution made this book possible. This project is based on a workshop<sup>2</sup> organised in 2013 at EURANDOM<sup>3</sup>, TU Eindhoven. The board thanks this institution for the kind hospitality and support. Furthermore, the board gratefully acknowledges the following programs that contributed to the funding of the meeting:

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- 3TU.AMI Applied Mathematics Institute, *Mathematics For Innovation*.

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<sup>2</sup><http://www.eurandom.nl/events/workshops/2013/PCA/PCA.html>

<sup>3</sup><http://www.eurandom.tue.nl/>

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**Pierre-Yves Louis** is an associate professor at the ‘Laboratoire de Mathématiques et Applications’ (University of Poitiers and CNRS, UMR 7348, France), where he teaches applied probability, statistics and stochastic modelling. His research interests include interacting systems of stochastic processes and algorithms, and stochastic models from mathematical, statistical, computational and applied perspectives (in particular in biology and medicine). He holds a PhD in mathematics from Université Lille 1 (France) and Politecnico di Milano (Italy), and his dissertation focused on a study of probabilistic cellular automata from a stochastic processes and statistical mechanics viewpoint. He previously served as an assistant at the University of Potsdam and post-doc fellow at the TU Berlin (Germany). He was trained in both pure mathematics and physics at the University Lille 1. While he was visiting scholar at Eurandom (TU Eindhoven and CNRS-UMI 3022), he was the main organiser of a workshop dedicated to PCA. P.-Y. Louis is a regularly invited researcher in the Netherlands, Germany and Italy, where he maintains several ongoing research collaborations. He supervises master students and PhD candidates.

**Francesca R. Nardi** is an associate professor at the University of Florence’s Department of Mathematics and Computer Science since 2016. Previously, she had a tenure track position (2006–2016) in the Stochastics Section of the Department of Mathematics and Computer Science at Eindhoven University of Technology (the Netherlands). She received her MSc degree and PhD degree in mathematics from the University of Rome “Tor Vergata”, Italy, and subsequently held post-doc and teaching positions at Eurandom (Eindhoven University of Technology), the University of Rome “La Sapienza”, and the University of Rome “Roma tre”. Her main research interests are in probability theory, statistical mechanics, and mathematical physics. Applications that motivate her research are statistical physics of equilibrium: phase diagrams and phase transitions; the statistical physics of non-equilibrium: meta-stability; the spread of epidemics over networks for moving populations; random walks in random environments; cut-off phenomena and random graphs. Francesca was the recipient (as PI) of a STAR grant and an Aspasia grant from the Netherlands Organization for Scientific Research (NWO). Francesca is regularly visiting The Netherlands where she has several ongoing projects and research collaboration. Francesca has more than 15 years experience teaching courses in basic probability, stochastic processes, large deviations, metastability and random graphs. She supervised and coached master students, PhD students and post docs.

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