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An Introduction to the Mathematical Theory of Dynamic Materials

Second Edition

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To the memory of Ella

Preface

The active media investigated for many decades in various fields are unified in a general theoretical concept of *dynamic materials* (DM), i.e., the substances with properties that vary in space and time. The term “materials” is perceived here in generalized sense: it equally applies to real material assemblages, as well as to environmental systems. The universal feature, common to all such implementations, is that they are thermodynamically open, i.e., they exist only due to the presence of non-interrupted exchange of mass/momentum/energy with the surrounding environment. One may briefly define DM as a conventional material framework *plus* the flux of said physical quantities into or away from it.

Progressive understanding of this concept originated in the 1990s through the study of optimal material design in dynamics. It has then been realized that the presence of time is necessary for such design as a factor that makes material layout a spatiotemporal entity capable of properly responding to challenges produced by a time-variable environment. Such formations belong to Minkowskian space, contrary to ordinary “dead” substances residing in Euclidean space. The DM concept therefore appears to be conceptually relativistic, though it is certainly not true that it should necessarily deal with relativistic material velocities.

These ideas constituted the framework of the first edition of this book dated back to 2007. Through the years since then, a number of new results opened the road toward better understanding of unusual effects hidden in spatiotemporal material geometry. Particularly, this fresh glance revealed a conceptually new mechanism of relaxation of material optimization problems in dynamics; this mechanism has released additional resources for optimization previously concealed in the property layouts.

A new version of the book also re-evaluates the role played by homogenization as a part of relaxation procedures. Remaining fundamental in statics,

this concept demonstrates limited significance in dynamics. To illustrate this, an interested reader is referred to examples in Chapters 5 and 6.

A new edition specifically concentrates on the differences between material optimization techniques in statics and dynamics. A better understanding of these differences is one of the goals pursued in the revised version. Though most of the text has been focused on systems with one spatial coordinate and time, the specifics of temporal property change reveals itself very clearly in this setting and prompts the ways toward forthcoming extensions and technical improvements.

As before, I enjoyed daily communion with my friends and colleagues Suzanne L. Weekes, Dan Onofrei, Mihhail Berezovski, Vadim Yakovlev, and William Sanguinet. Slava Krylov has introduced me into the exciting field of MEMS and NEMS that currently form a technological basis for a real-life implementation of DM. The text of Section 1.3.1, Chapter 1, has been written by Slava and is included into the book with his generous permission.

An invaluable help of Ellen M. Mackin and William C. Sanguinet has made the work over the manuscript much easier for me. It is a special pleasure to express my gratitude to my colleagues for their friendly assistance.

Worcester, MA, USA
May 2017

Konstantin A. Lurie

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