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Linear Algebra

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To my daughter NOUR

Preface

Linear algebra is the study of the algebraic properties of linear transformations and matrices and it is an essential part of virtually all areas of mathematics. It is also a fundamental and an extremely powerful tool in every single discipline of sciences and engineering.

This is a self-contained textbook on linear algebra written in an easy way, so that it can be accessible to many readers. It begins in ▶ Chap. 1 with the simplest linear equation and generalizes many notions about this equation to systems of linear equations, and then introduces the main ideas using matrices and their properties. We believe that this is the right approach, since most students take the first course of linear algebra already knowing something about linear equations, lines, and systems of linear equations. Then follows a detailed chapter (▶ Chap. 2) on determinants and their properties where we also study the relationship between determinants and the inverses of matrices and the use of determinants in solving systems of linear equations. We introduce the main ideas with detailed proofs. We also investigate some particular determinants that are very useful in applications. In addition, we explain in a simple way where the ideas of determinants come from and how they fit together in the whole theory.

In ▶ Chap. 3, we introduce the Euclidean spaces using very simple geometric ideas and then discuss various important inequalities and identities. These ideas are present in the theory of general Hilbert spaces in a course of functional analysis, so it is much better for students to learn them and understand them clearly in Euclidean spaces.

The core of ▶ Chap. 4 is a detailed discussion of general vector spaces where rigorous proofs to all the main results in this book are given. This is followed by a chapter (▶ Chap. 5) on linear transformations and their properties.

In ▶ Chap. 6, we introduce notions concerning matrices through linear transformations, trying to bridge the gap between matrix theory and linear algebra.

▶ Chapters 7 and 8 are more advanced, where we introduce all the necessary ideas concerning eigenvalues and eigenvectors and the theory of symmetric and orthogonal matrices.

One of the aspects that should make this textbook useful for students is the presence of exercises at the end of each chapter. We did choose these exercises very carefully to illustrate the main ideas. Since some of them are taken (with some modifications) from recently published papers, it is possible that these exercises appear for the first time in a textbook. All the exercises are provided with detailed solutions and in each solution, we refer to the main theorems in the text when necessary, so students can see the main tools used in the solution. In addition all the main ideas in this book come with illustrating examples. We did strive to choose solutions and proofs that are elegant and short. We also tried to make this textbook in about 400 pages by focusing on the main ideas, so that students will be able to easily and quickly understand things. In addition, we tried to maintain a balance between the theory of matrices and the one of vector spaces and linear transformations.

This book can be used as a textbook for a first course in linear algebra for undergraduate students in all disciplines. It can be also used as a booklet for graduate students, allowing to acquire some concepts, examples, and basic results. It is also suitable for those students who are looking for simple, easy, and clear textbook that summarizes the main ideas of linear algebra. Finally it is also intended for those students who are interested in rigorous proofs of the main theorems in linear algebra. We believe that if a good student uses this book, then she (he) can read and learn the basics of linear algebra on her (his) own.

We would like to thank Salim A. Messaoudi (from KFUPM) for valuable suggestions and corrections which improved the contents of some parts of this book. We also thank Sofiane Bouarroudj (from NYU Abu Dhabi) for the many discussions that we have had about the proofs of some theorems of linear algebra.

Abu Dhabi, United Arab Emirates Belkacem Said-Houari
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Contents

List of Figures