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The Anomalous Magnetic Moment of the Muon

Second Edition

Springer

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The closer you look the more there is to see

Preface to the Second Edition

Ten years later, where are we? Why a second edition? The two next generation muon $g-2$ experiments at Fermilab in the US and at J-PARC in Japan have been designed to reach a four times better precision of 16×10^{-11} (from 0.54 ppm to 0.14 ppm) and the challenge for the theory side is to keep up in precision if possible. This has triggered a lot of new research activities which justify an update of the first edition. The main motivation is the persisting 3 to 4 σ deviation between standard theory and experiment. As Standard Model predictions almost without exception match perfectly all experimental information, the deviation in one of the most precisely measured quantities in particle physics remains a mystery and inspires the imagination of model builders. A flush of speculations are aiming to explain what beyond the Standard Model effects could fill the gap. Here very high precision experiments are competing with searches for new physics at the high energy frontier set by the Large Hadron Collider at CERN. Actually, the tension is increasing day-by-day as no new states are found which could accommodate the $(g_\mu - 2)$ discrepancy. With the new muon $g-2$ experiments this discrepancy would go up at least to 6 to 10 σ, in case the central values do not move, the 10 σ could be reached if the present theory error could be reduced by a factor of two.

The anomalous magnetic moment of the muon is a number represented by on overlay of a large number of individual quantum corrections, which depend on a few fundamental parameters. An update of the latter actually changes almost all numbers in the last digits. Besides this, there has been remarkable progress in the calculation of the higher order corrections. Aoyama, Hayakawa, Kinoshita and Nio managed to evaluate the five-loop QED correction, which includes about 13 000 diagrams and which account for a small 5×10^{-11}, thereby reducing the uncertainty of the QED part which has been dominated by the missing $O(\alpha^5)$ correction. More recently a seminal article by Laporta the essentially exact universal 4-loop contribution has been presented. The corresponding contributions to the electron $g-2$ together with the extremely precise determination of $(g_e - 2)$ by Gabrielse et al. allows one to determine a more precise value of the fine structure constant α, which in turn affect the numbers predicted for $(g_\mu - 2)$. Also more precise lepton mass ratios recommended by the CODATA group are slightly affecting the predictions. To the weak interaction contribution the uncertainty could be reduced mainly because of the fact that after the discovery of the Higgs particle by ATLAS and CMS at the Large Hadron Collider at CERN, the last relevant missing Standard Model parameter could be determined with remarkable precision.

Still the largest uncertainties in the SM prediction come from the leading hadronic contributions: the hadronic vacuum polarization and the hadronic light-by-light scattering insertions. The hadronic vacuum polarization at $O(\alpha^2)$, evaluated in terms of e^+e^--annihilation data via a dispersion relation has been improved substantially mainly with new data from initial state radiation approach that the Φ factory DAFNE at Frascati with the KLOE detector and at the B factory at SLAC with the BaBar detector. Lately also new results from BEPC-II at Beijing with the BES-III detector and from VEPP-2000 at Novosibirsk with the CMD-3 and SND detectors contributed to further reduce the uncertainties. On the theory side the τ-decay spectra versus e^+e^--annihilation data which should essentially agree after an isospin rotation has been resolved by including missing $\gamma - \rho^0$ mixing effects. Besides the NLO vacuum polarization new the NNLO amounting to 12×10^{-11} roughly a 1 σ effects has been calculated by Kurz et al. recently. In the meantime also non-perturbative ab initio lattice QCD calculations come closer to be competitive with the e^+e^--data based approach. I therefore included an introduction to the lattice QCD approach at the end of Chap. 5. The activity here has been dramatically developed. While ten years ago there has been essentially one group only active, now there are a least six groups competing.

The most challenging problem remains the hadronic light-by-light contribution of $O(\alpha^3)$. Unlike the hadronic vacuum polarization which is a one scale problem, the hadronic light-by-light scattering involves three different scales and there are many different hadronic channels contributing. The only fairly complete calculations are based on low energy effective hadronic models, which unfortunately sill are not constrained by data to a satisfactory degree. Quite recently, a new approach has been worked out by Colangelo, Hoferichter, Procura and Stoffer, and Pauk and Vanderhaeghen which attempts to rely completely on hadronic light-by-light scattering data in conjunction with dispersion relations. This sounds to implement the successful hadronic vacuum polarization technique to the multi channel multi scale light-by-light case. Apart from being much more elaborate the data pool is by far not as complete as in the e^+e^- data case. In spite of the fact that data for a complete evaluation are largely missing there is definitely progress possible with exploiting existing data for $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ in particular, where new data from Belle are of good quality, which allows one to get more solid evaluations than existing ones. For the singly tagged pion transition form factor there have been new useful data from BaBar and Belle which cover a much larger energy range now. Also in this case lattice QCD starts to be a new player in the field, and first useful information concerning the doubly tagged pion transition form factor has been evaluated and provides an important new constraint.

The main focus of the book is a detailed account of the Standard Model prediction.

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I gratefully acknowledge the support of our topical workshop[1] on *Hadronic contributions to the muon anomalous magnetic moment: strategies for improvements of the accuracy of the theoretical prediction,* 1–5 April 2014, Waldthausen Castle near Mainz, by the PRISMA Cluster of Excellence, the Mainz Institute for Theoretical Physics MITP, and the Deutsche Forschungsgemeinschaft DFG through the Collaborative Research Center *The Low-Energy Frontier of the Standard Model* (SFB 1044).

Wildau/Berlin, Germany Friedrich Jegerlehner
March 2017

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[1] Organized by: Tom Blum, Univ. of Connecticut; Simon Eidelman, INP Novosibirsk; Fred Jegerlehner, HU Berlin; Dominik Stöckinger, TU Dresden; Achim Denig, Marc Vanderhaeghen, JGU Mainz (see https://indico.mitp.uni-mainz.de/event/13/).

Preface to the First Edition

It seems to be a strange enterprise to attempt write a physics book about a single number. It was not my idea to do so, but why not. In mathematics, maybe, one would write a book about π. Certainly, the muon's anomalous magnetic moment is a very special number and today reflects almost the full spectrum of effects incorporated in today's Standard Model (SM) of fundamental interactions, including the electromagnetic, the weak and the strong forces. The muon $g-2$, how it is also called, is a truly fascinating theme both from an experimental and from a theoretical point of view and it has played a crucial role in the development of QED which finally developed into the SM by successive inclusion of the weak and the strong interactions. The topic has fascinated a large number of particle physicists last but not least it was always a benchmark for theory as a monitor for effects beyond what was known at the time. As an example, nobody could believe that a muon is just a heavy version of an electron, why should nature repeat itself, it hardly can make sense. The first precise muon $g-2$ experiment at CERN answered that question: yes the muon is just a heavier replica of the electron! Today we know we have a 3-fold replica world, there exist three families of leptons, neutrinos, up-quarks and down-quarks, and we know we need them to get in a way for free a tiny breaking at the per mill level of the fundamental symmetry of time-reversal invariance, by a phase in the family mixing matrix. At least three families must be there to allow for this possibility. This symmetry breaking also know as CP-violation is mandatory for the existence of all normal matter in our universe which clustered into galaxies, stars, planets, and after all allowed life to develop. Actually, this observed matter-antimatter asymmetry, to our present knowledge, cries for additional CP violating interactions, beyond what is exhibited in the SM. And maybe it is a_μ which already gives us a hint how such a basic problem could find its solution. The muon was the first replica particle found. At the time, the existence of the muon surprised physicists so much that the Nobel laureate Isidor I. Rabi exclaimed, "Who ordered that?". But the muon is special in many other respects and its unique properties allow us to play experiment and theory to the extreme in precision.

One of the key points of the anomalous magnetic moment is its simplicity as an observable. It has a classical static meaning while at the same time it is a highly non-trivial quantity reflecting the quantum structure of nature in many facets. This simplicity goes along with an unambiguous definition and a well understood quasi classical behavior in a static perfectly homogeneous magnetic field. At the same time the anomalous magnetic moment is tricky to calculate in particular if one wants to know it precisely. To start with, the problem is the same as for the electron, and how tricky it was one may anticipate if one considers the 20 years it took for the most clever people of the time to go form Dirac's prediction of the gyromagnetic ratio $g = 2$ to the anomalous $g - 2 = \alpha/\pi$ of Schwinger.

Today the single number $a_\mu = (g_\mu - 2)/2$ in fact is an overlay of truly many numbers, in a sense hundreds or thousands (as many as there are Feynman diagrams contributing), of different signs and sizes and only if each of these numbers is calculated with sufficient accuracy the correct answer can be obtained; if one single significant contribution fails to be correct also our single number ceases to have any meaning beyond that wrong digit. So high accuracy is the requirement and challenge.

For the unstable short lived muon which decays after about 2 micro seconds, for a long time nobody knew how one could measure its anomalous magnetic moment. Only when parity violation was discovered by end of the 1950's one immediately realized how to polarize muons and how to study the motion of the spin in a magnetic field and to measure the Larmor precession frequency which allows to extract a_μ. The muon $g - 2$ is very special, it is in many respect much more interesting than the electron $g - 2$, and the $g - 2$ of the τ, for example, we are not even able to confirm that $g_\tau \sim 2$ because the τ is by far too short lived to allow for a measurement of its anomaly with presently available technology. So the muon is a real lucky case as a probe for investigating physics at the frontier of our knowledge. By now, with the advent of the recent muon $g - 2$ experiment, performed at Brookhaven National Laboratory with an unprecedented precision of 0.54 parts per million, the anomalous magnetic moment of the muon is not only one of the most precisely measured quantities in particle physics, but theory and experiment lie apart by three standard deviations, the biggest “discrepancy” among all well measured and understood precision observables at present.

This promises nearby new physics, which future accelerator experiments are certainly going to disentangle. It may indicate that we are at the beginning of a new understanding of fundamental physics beyond or behind the SM. Note however, that this is a small deviation and usually a 5 standard deviation is required to be accepted as a real deviation, i.e. there is a small chance that the gap is a statistical fluctuation only.

One would expect that it is very easy to invent new particles and/or interactions to account for the missing contribution from the theory side. Surprisingly other experimental constraints, in particular the absence of any other real deviation from the SM makes it hard to find a simple explanation. Most remarkable, in spite of these tensions between different experiments, the minimal supersymmetric extension of the standard model, which promised new physics to be “around the corner”, is precisely what could fit. So the presently observed deviation in $g-2$ of the muon feeds hopes that the end of the SM is in sight.

About the book: in view of the fact that there now exist a number of excellent more or less extended reviews, rather than adding another topical report, I tried to write a self-contained book not only about the status of the present knowledge on the anomalous magnetic moment of the muon, but also remembering the reader about its basic context and its role it played in developing the basic theoretical framework of particle theory. After all, the triumph this scientific achievement marks, for both theory and experiment, has its feedback on its roots as it ever had in the past. I hope it makes the book more accessible for non-experts and it is the goal to reach a broader community to learn about this interesting topic without compromising with respect to provide a basic understanding of what it means.

So the books is addressed to graduate students and experimenters interested in deepening some theoretical background and to learn in some detail how it really works. Thus, the book is not primarily addressed to the experts, but nevertheless gives an up-to-date status report on the topic. Knowledge of special relativity and quantum mechanics and a previous encounter with QED are expected.

While the structural background of theory is indispensable for putting into perspective its fundamental aspects, it is in the nature of the theme that numbers and the comparison with the experiment play a key role in this book.

The book is organized as follows: Part I presents a brief history of the subject followed in Chap. 2 by an outline of the concepts of quantum field theory and an introduction into QED including one-loop renormalization and a calculation of the leading lepton anomaly as well as some tools like the renormalization group, scalar QED for pions and a sketch of QCD. Chapter 3 first discusses the motion of leptons in an external field in the classical limit and then overviews the profile of the physics which comes into play and what is the status for the electron and the muon $g-2$’s. The basic concept and tools for calculating higher order effects are outlined.

In Part II the contributions to the muon $g-2$ are discussed in detail. Chapter 4 reviews the QED calculations. Chapter 5 is devoted to the hadronic contributions in particular to the problems of evaluating the leading vacuum polarization contributions from electron-positron annihilation data. Also hadronic light-by-light scattering is critically reviewed. Chapter 6 describes the principle of the experiment in some detail as well as some other background relevant for determining $g_\mu - 2$. The final Chap. 7 gives a detailed comparison of theory with the experiment and discusses possible impact for physics beyond the standard theory and future perspectives.

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Friedrich Jegerlehner

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