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Stable Non-Gaussian Self-Similar Processes with Stationary Increments

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To Terese, Jovita, and Milda
and
to Jeremy and Yael

Preface

Fractional Brownian motion is (up to a constant and for a fixed self-similarity parameter) the unique Gaussian self-similar process with stationary increments. When the assumption of Gaussian distributions is replaced by that of stable (non-Gaussian) distributions, the situation is more complex. This is because there are in fact many different such stable (non-Gaussian) processes (this is for the same self-similarity and stability parameters, discounting multiplicative constants). This work provides a self-contained presentation on the structure of a large class of these stable processes, known as *self-similar mixed moving averages*. These include

- Linear fractional stable motion (LFSM)
- Log-fractional stable motion
- Mixed truncated fractional stable motion
- The Samorodnitsky processes
- The Takenaka processes
- The Telecom process

All these processes are different extensions of fractional Brownian motion to the infinite variance stable case. They are defined through integral representations with respect to a stable non-Gaussian measure. *Minimal integral representations* are introduced first, and then the *rigidity properties of stable processes* are discussed. The rigidity will allow us to take advantage of invariance properties such as self-similarity. We will in fact relate stable processes with an invariance property, such as self-similar mixed moving averages, to nonsingular flows and their functionals. Various decompositions of flows are discussed, including

- Dissipative flows
- Conservative flows
- Periodic flows
- Cyclic flows
- Fixed (identity) flows

The periodic, cyclic, and fixed flows are typical examples of conservative flows. We also provide an example of "the fourth kind," namely a conservative flow which is not one of these. These flows are important because they lead to decompositions of the associated self-similar mixed moving averages in major components.

By using minimal representations and flows, we will be able to show that various processes such as those listed above are different from each other. Minimal representations can thus serve to identify the process but they are not always very easy to determine in practice. We will also provide identification criteria which do not rely on either minimal representations or flows, and which are based instead on the structure of the kernel function in the integral representation of the process.

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Acronyms

FBM	Fractional Brownian motion
CFSM	Conservative fractional stable motion
cLFSM	Cyclic fractional stable motion
(C\P)FSM	Conservative nonperiodic fractional stable motion
DFSM	Dissipative fractional stable motion
FFSM	Fixed fractional stable motion
LFSM	Linear fractional stable motion
PFSM	Periodic fractional stable motion
$S\alpha S$	Symmetric α-stable