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Editors

# Modeling, Analysis, and Visualization of Anisotropy

 Springer

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# Preface

Starting from the entire Universe and proceeding all the way down to the atomic scale, physical quantities often take on different values when observed along different directions. Directional dependence (anisotropy) of data is thus paramount to numerous scientific disciplines such as chemistry, material science, astrophysics, neuroscience, and medical imaging. In each of these disciplines, a wide array of works employs tensors and other mathematical constructs to represent anisotropy. The focus of this book is on the modeling, processing, and visualization of anisotropy, regardless of the context in which it emerges. As such, it differs substantially from conventional reference works, which tend to be centered on a particular application.

This multidisciplinary book is the sixth in a series that aims to foster scientific exchange between communities that employ tensors and other higher-order representations of directionally dependent data. A significant portion of the chapters were co-authored by the participants of the workshop titled “Multidisciplinary Approaches to Multivalued Data: Modeling, Visualization, Analysis,” which was held in Dagstuhl, Germany, in April 2016. However, the book does not gather the proceedings of the workshop; some contributions deviate from the presentations at the workshop, and we have invited several completely new chapters within the scope of the workshop.

The first four chapters of the book make up Part I, which focuses on characterizing the features of and visualizing tensor fields. In turn, Part II includes a collection of chapters on the processing of this data. Here, diffusion anisotropy continues to be a topic of interest. More fundamental aspects of the topic are addressed in Part III, while some important applications are presented in Part IV. Part V, which is the final part, highlights nascent approaches inspired by machine learning.

Visualization and analysis provide the necessary basis for understanding and exploring data, which is especially true and challenging when it comes to anisotropic or higher-order entities. Part I of this book presents different strategies for the structural analysis of tensor data, which serve as basis for condensed multiscale visualization. The methods range from topological analysis, to statistical moments, to the analysis of the gradient of tensors in materials. Chapter 1 starts with

theoretical considerations regarding the stability of topological structures with respect to perturbations in the tensor field. The result is a hierarchy of features that supports multiscale analysis. Chapter 2 provides a more applied perspective on tensor topology and its interpretations in the context of solid mechanics simulations. A different approach to structure is followed in Chap. 3, which uses moment invariants as descriptors of the tensor field. These descriptors are used to detect patterns in a data set independent of its orientation and scale. The last chapter of Part I investigates the gradients of (symmetric) second-order tensor fields, which are tensors of third order. Here, special emphasis is put on stress gradients in structural mechanics, for which glyph-based visualization is proposed.

Various imaging methods represent important sources of anisotropic data and produce voxelized data or unstructured point clouds. The analysis and processing of such data are the topic of Part II of this book. The first step in analyzing discrete data is the choice of an appropriate interpolation schema, which is the topic of Chap. 5. It provides a survey of interpolation schemes, including a detailed analysis of their respective properties. Chapter 6 proposes a general framework for fundamental morphological operations, such as dilation and erosion for matrix fields. The next two chapters deal with the analysis of point clouds: Chap. 7 proposes a robust extraction of geometrical shapes like curves and surfaces from point clouds, even in noisy scenarios, and introduces an advanced tensor voting technique for blood vessel analysis. Chapter 8, which is the last chapter, uses a feature classification approach to extract and reconstruct the point-sampled geometry of topographic data captured using airborne LiDAR technology.

Measuring structural anisotropy within locally oriented media is a challenging problem with enormous implications for assessing the state of neural tissues. Magnetic resonance techniques provide a powerful though indirect means of noninvasively measuring such anisotropy by characterizing the orientational preference of water diffusion. In Chap. 9 of Part III, the authors illustrate how anisotropic information can be obtained via diffusion MR. Various anisotropy metrics obtained through state-of-the-art signal-based and multicompartmental models are reviewed and compared. The rather large voxels afforded by typical MR imaging studies complicate the determination of local (microscopic) anisotropy when there is significant heterogeneity within the voxel. In a subsequent chapter, the notion of microscopic anisotropy and recent developments concerning its measurement via traditional as well as novel MR pulse sequences are reviewed. In Chap. 11, estimation of the single-diffusion tensor model in each voxel is revisited from a Bayesian perspective by incorporating heteroscedastic noise (i.e., the variance does not have to be constant but can also change throughout the samples). In the last chapter, intra-voxel heterogeneity is modeled by means of a multicompartmental model for diffusion MRI, employing a combination of tensor-valued non-central Wishart distributions.

Diffusion MRI fiber tractography remains the only available technique for mapping the brain's structural connectivity *in vivo*. The diverse contributions in Part IV demonstrate that it also remains a vital and active research topic. Chapter 13 introduces a novel method for obtaining a geometric representation of

tract boundaries from streamline-based tractography via a curve similarity metric, while Chap. 14 contributes to the validation of two widely used software packages for fiber tractography by investigating their test-retest reliability. Chapter 15, which is the final chapter, presents a pipeline for automated fiber bundle segmentation and quantification, which is specifically targeted toward the assessment of brain maturity in preterm neonates.

In recent years, the broad use of deep learning approaches has significantly enhanced the state of the art in neighboring fields, such as computer vision. However, there has been relatively little work on adapting such techniques to tensor fields or other mathematical representations of anisotropy. Accordingly, Part V of our book includes two contributions addressing this timely field of research. The first explores the feasibility of using a convolutional neural network (CNN) to map strain tensors to an associated scalar value, while the second uses CNNs to reconstruct fiber orientation distribution functions from diffusion MRI, which continues to work well even when the number of measurements provided as input is reduced.

We wish to thank the board and staff at Schloss Dagstuhl for their time and effort in facilitating the workshop, which fostered scientific exchange and ultimately led to the creation of this book. Special thanks go to the editors of the Springer Mathematics and Visualization book series for their valued consideration and support. The book would of course never have been possible without the high-quality manuscripts submitted by the authors. Last but not least, we are indebted to all reviewers, whose incisive comments greatly improved many of the chapters.

We certainly hope that this book will be a valuable resource for those who work on multidirectional data and inspirational in the development of new models as well as analysis and visualization techniques, thus advancing the state of the art in studies involving anisotropy.

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March 2017

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