Durability of Springs

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Foreword

Technical springs are well-known machine components, which can be reversibly deformed under load and also under cyclical or oscillating forces. Springs transform kinetic energy into potential energy, store energy, and feed it back nearly without loss into the system when relieved. To use these features for optimized applications, two essential issues have to be considered: the characteristic of the used material as well as an adapted and optimized spring design for the application. Spring steel alloys have the matching material characteristic for the springs, which are mostly highly stressed. Furthermore, the well-calculated shape of the spring allows fulfilling the technical requirements and characteristics like stiffness, fatigue life, and so on. Different spring designs are classified according to their shape as well as to their type of load stresses, which gives in most cases also the basic understanding for their technical calculation. Due to their multiple technical characteristics and functions, springs are still nearly irreplaceable components in any new and modern machine concept, in planes, ships, buildings, trains, or automobiles. To fulfill all those high demands, standards, and specifications, accurate calculation methods are required, with an approach for all important physical effects for springs.

The purpose of the book is to explain the mechanical and physical properties of specific steel alloy springs and to present supplementary analytical calculation methods based on already existing and summarized calculation models. Approaches for characteristic spring data like weight and package, lifetime and crack growth, creeping and relaxation rate as well as transverse vibrations and natural frequencies are shown for specific spring shapes. The book contains calculations for helical springs, disc springs, wave springs, and thin walled rods with a semi-opened cross section. Due to the analytical approach of all calculation models, ambitious development engineers and design engineers get a helpful review and overview of existing and supplementary calculation methods for springs.

Prof. Vladimir Kobelev was born in Rostow-na Donu, Russian Federation. He studied Physical Engineering at the Moscow Institute of Physics and Technology. After his PhD from the Department of Aerophysics and Space Research (FAKI), he habilitated at the University of Siegen, Scientific-Technical Faculty. Today, Prof.

Kobelev is lecturer and APL professor at the University of Siegen in the subject area of Mechanical Engineering.

In his industrial career, Prof. Kobelev is an employee at Mubea, a successful automotive supplier located near Cologne/Germany. In the Corporate Engineering Department, Prof. Kobelev is responsible for the development of calculation methods and physical modeling of Mubea components.

Muhr and Bender KG (Mubea) Dr.-Ing. Dr.-Ing. E. h. Thomas Muhr
Attendorn, Germany

Preface

One of the oldest elements in machines is the technical spring. Their applications are as varied as the developers' ideas. While working, the components are mostly concealed, almost invisible, and are seldom noticed at all. But this construction part is not to be underestimated. It does its job, as a safety element in brakes, or as a comfort element in the chassis. Without a valve spring no motor could run, and without a spring no lock could be opened or closed. These are just some exemplary applications of the often hidden helpers. At first glance, springs appear simply trivial and ubiquitous. However, on closer examination it must be admitted that there is far more behind the spring than most of us realise. The demands on the component are increasing more and more. While in the past the simple relationship between force and distance, Hooke's Law, was sufficient, today complex regulations about the load and environmental conditions, durability, and weight reduction have become standard. Successful research has been carried out for many years in the field of springs. Much of the knowledge collected has been included in this book. In this work, developers have the opportunity to gain detailed knowledge of springs. Prof. Dr. Kobelev has provided a comprehensive high-level insight into the world of spring development and thus created a solid basis for the design and engineering of springs. The relationship between the physics of the material and the mechanical load on the part is explained.

I would like to wish readers success in their involvement with this fascinating topic:

"The Durability of Springs"

Hagen, Germany — Wolfgang Hermann

Introduction

The integral parts of many mechanical systems are elastic elements or springs. The spring is the widespread resilient element which is used in the industrial machinery and automotive systems, as diesel fuel pumps, valve trains, brakes, suspensions, seats, doors, and control elements. For reducing impact events in some heavy trucks and railroad cars primarily, helical, or coil, springs are applied. In some vehicles, torsion bars are used instead of the coil springs. The reduction of weight of the suspension springs causes the decrease of unsprung mass of the axle and has a positive influence on the comfort, traction, and steering properties of the car. The development of modern passenger cars has highlighted a trend toward reduced package space for suspension components in order to maximize package space for occupants and loads. Such requirements lead to reduction in spring dimensions and wire cross section. Springs can be found in high-precision testing devices, in which springs play the role of energy harvesters. The efficient design procedures for spring elements are based on the modern simulation and optimization methods.

The springs make possible to maintain a tension or a force in a mechanical system, to absorb the shocks, and to reduce the vibrations. A fatigue failure of flexural elements often causes the damage of the complete machinery component and provokes high costs. The high-loaded spring elements in modern industrial equipment and transportation must survive a very high number of cycles with high mean stress as well as high amplitude stress. These springs are manufactured of qualitative wires and by means of distinctive mechanical and heat treatment processes.

Helical springs are formed by wrapping wire or rod of uniform cross section around a cylinder. A fixed distance between the successive coils of a spring is maintained, so that the axis of the wire forms a helix. The standard design procedures for helical springs are described in Spring Design Manual 1996 (DIN 2012, 2013, 2015). The springs are generally produced from oil-tempered steel wire, which is wire formed by drawing hot rolled steel rod through a drawing die and oil-tempering the resultant wire. Oil-tempering is a term of art identifying a process generally involving heating the wire to austenitization temperatures, quenching it in oil, tempering it by heating it, and recoiling it. This sequence of manufacturing steps increases the ultimate stress of the material. However, the ductility of the austenitized material reduces. The material behaves almost elastically up to the moment of breakage. The influences of both effects, namely, demand to increase the ultimate stress and decrease of ductility, on the fatigue life of material are conflicting.

The setting reduces the relaxation and improves the creep behavior of springs at operating temperature. It is well known that the setting influences the static residual stresses in the spring and changes the cyclic fatigue properties of the spring. The cold-setting and heat-setting procedures are used in modern spring manufacturing. Heat setting designates the production step of time-depending loading of spring at elevated temperature. The main physical process during spring setting is the creep of material.

Shot peening is a mechanical surface treatment that is used to improve spring performance. The local plastic deformation on the wire surface occurs, which leads to an enhancement and strengthening of properly machined surface. Shot peening considerably increases the fatigue life of springs.

The book presents the theory of elastic elements from the point of classical mechanics. The book studies most important problems, which are necessary for understanding of manufacturing process and behavior of spring elements. What all considered problems have in common is that they are solved in closed form. The elements of creep, plasticity, and fatigue serve as the building blocks of physical background.

The optimization of springs is studied in Chap. 1. The design formulas for linear helical springs with an inconstant wire diameter and with a variable mean diameter of spring are presented. Based on these formulas, the optimization of spring for given spring rate and strength of the wire is performed. The basic design principles for optimal leaf springs are also discussed.

The torsion problems for straight cylinders with circular and elliptical cross sections allow the well-known closed form solutions. Chapter 2 presents analytical solutions for the torsion problem of an incomplete torus with circular and non-circular cross sections. The pitch of helix is ignored. The hollow cross sections of the particular form also demonstrate a closed form of analytical solution. The solution is useful for the analysis and design of helical springs with non-circular wires.

Chapter 3 explains a powerful method for the simplification of helical spring equations. Instead of treatment of helical spiral wire, the deformation of the virtual middle line is studied. The virtual middle line is provided with the extension, torsion, and bending stiffness and behaves as an initially straight elastic rod or column. This simplification allows uncomplicated solutions of several practically important problems. For explanation, the load dependence of transverse vibrations for helical springs and the transformation of transversal vibration to buckling mode are addressed. The lateral buckling of spring is considered in the framework of dynamic stability as the limit case for the vibration analysis.

At the beginning, the equations for transverse vibrations of the axially loaded linear helical springs are developed. The method is based on the traditional concept of an equivalent column. Secondly, one reveals the effect of axial load on the fundamental frequency of transverse vibrations and derives the explicit formulas for this frequency. It is shown that the fundamental natural frequency of the transverse vibrations of the spring depends on the variable length of the spring. The predominant reduction of frequency with the shortened length of the spring is demonstrated. Finally, when the frequency nullifies, the side buckling spring by divergence mode occurs.

For proper accounting of dynamic effects, the models of flexible springs with massive wire are required. In some cases, such as when the spring is uniform, analytical models for dynamics and buckling can be developed. However, in typical springs, only the central turns are uniform; the ends are often not (e.g., having a varying helix angle or cross section). Thus, obtaining analytical models in this case can be very difficult if at all possible. A variety of theories to describe the dynamic behavior of helical springs, which involves interaction of bending (flexural), torsion, and longitudinal waves, can be found in the literature. Alongside this, various approximate methods are employed to determine the fundamental frequencies of vibrations of springs. One can roughly divide the methods used to determine the fundamental frequencies in three groups:

Analysis methods, based on the concept of an equivalent column
Exact analysis methods, based on the theory of spatially curved bars
Numerical methods, based on finite-element formulation for spatially curved bars

Mechanical problems arising during the manufacturing of helical springs are examined in Chap. 4. The plasticization process and appearance residual stress is studied. The plastic analysis of spring coiling of helical springs is performed. It is well known that the excessive stresses during the coiling of helical springs could lead to breakage of the rod. Moreover, the high level of residual stress in the formed helical spring reduces considerably its fatigue life. For the practical estimation of residual and coiling stresses in the helical springs, the analytical formulas are necessary. In this chapter, the analytical solution of the problem of elastic-plastic deformation of cylindrical bar under combined bending and torsion moments is found for a special nonlinear stress–strain law. The obtained solution allows the analysis of the active stresses during the combined bending and twist. Additionally, the residual stresses in the bar after spring-back are also derived in closed analytical form. The obtained results match the reported measured values. The developed method does not require numerical simulation and is perfectly suited for programming of coiling machines, for estimation of loads during manufacturing of cold-wounded helical springs, and for dimensioning and wear calculation of coiling tools.

Disk springs (also known as Belleville washers) are studied in Chap. 5. The disk springs are shallow conical rings that are subjected to axial loads. Normally, the ring thickness is constant and the applied load is evenly distributed over the upper inside edge and lower outside edge. Disk springs are generally manufactured from spring steel and can be subjected to static loads, rarely alternating loads, and dynamic loads. Disk springs can satisfy the most severe fatigue life and set loss requirements. In this chapter, the equations of equilibrium of disk springs of thin and moderate thickness are obtained through the variational principles for conical shells. The closed form analytical solutions based on the common deformation hypotheses for the equations of thin and thick truncated conical shells are achieved.

In Chap. 5, disk wave springs are also analyzed. Both linear and nonlinear disk wave springs are discussed.

The understanding of the behavior of springs under high static load is essential for their correct design. The creep and relaxation of springs is the subject of Chap. 6. Stress analysis for creep has a long history in engineering mechanics driven by the requests of design for elevated temperature. The examples of high loaded elements of machinery deliver the springs made of steel. Steel springs are the typical energy storing elements of valve train in engines, clutches, and automatic transmissions of cars. The coned-disk spring, Belleville spring or cupped spring washer, or Belleville washers are typically used as springs, or to apply a pre-load or flexible quality to a bolted joint or bearing. As the basic properties of Belleville washers include high fatigue life, better space utilization, low creep tendency, and high load capacity with a small spring deflection. The physical phenomenon with metal springs is that at stress below the yield strength of the material a slow inelastic deformation takes place. In the spring branch, this is called creep when a spring under constant load loses length, and it is called relaxation when a spring under constant compression loses load. The creep and relaxation rates depend on the temperature, the stress in the metal, the yield strength, and the time. Increased temperature, stress, and time also increase the creep and relaxation rates. Especially the temperature and stress have a major influence. The precise creep description is essentially important for correct dimensioning of springs. Finally, Chap. 6 demonstrates the evaluation of creep constants in a wire twist experiment.

The aim of the Chap. 7 is to derive the exact analytical expressions for torsion and bending creep of rods with the common and fractional Norton-Bailey constitutive models. This fractional constitutive model is based on adaptations for time-varying stress of equally simple models for the secondary creep stage. The common secondary creep constitutive model has been the Norton-Bailey Law which gives a power law relationship between minimum creep rate and (constant) stress. The exact analytical expressions giving the torque and bending moment as a function of the time were derived for these nonlinear creep laws. The distinctive mathematical properties of the power law allowed the development of analytical methods, many of which can be found in high-temperature design codes. In Chap. 7, the generalized expression for creep law is studied. The new expression is based on the experimental data and unifies the primary, secondary, and tertiary regions of creep curve. The relaxation functions for bending and torsion depend only on the maximal stress in the cross section, which occurs on the outer surface of the coil.

The durability of spring under high oscillation loads is the subject of Chap. 8. Traditional methods of fatigue design are based on the acquisition of numerous experimental data in cyclic tests, data structuring, and extraction of empirical formulas. The new method for analysis of crack growth under repeated load is introduced in Chap. 8. The proposed method starts from the micromechanically inspired effects of crack propagation, explains the history of crack spreading, and finally delivers the stress–life curves. The expressions for spring length over the number of cycles are derived in terms of higher transcendental function. The closed form solutions are used for the estimation of the fatigue life of heterogeneously stressed structural members.

The developed theory is applied in Chap. 9 to helical springs under cyclic load. The probability distribution of the fatigue limit for heterogeneously stressed structural elements is evaluated. The proposed approach for the stress gradient sensitivity of fatigue life is based on the weakest link concept. The weakest link approach is applied to calculate the number of cycles to crack initiation of structural elements under different probability levels. The effect of stress ratio on elongation of crack is discussed.

The fatigue sensitivity to stress concentration is addressed in application to springs. Effect of fluctuating stresses on fatigue life of springs is combined with the influence of heterogeneous stress distribution (stress gradient) over the cross section of wire and time-varying stresses. These two factors lead to complicated evaluation for fatigue life of helical springs. The stress field is inhomogeneous over the cross section of the wire of spring. The stress distribution is uniquely defined by ratio of the diameter of wire to the diameter of spring body. The calculated lifetimes are compared with the lifetimes obtained from experiments performed on helical springs subjected to cyclic load of constant amplitude.

The analysis of thin-walled rods with semi-opened cross section is performed in Chap. 10. An essential characteristic for this class of thin-walled beam-like structures is their closed but flattened profile. In this book, an intermediate class of thin-walled beam cross sections is studied. The cross section of the beam is closed, but the shape of cross section is elongated and curved. The walls, which form the cross section, are nearly equidistant. The unusual shape of semi-opened thin-walled beams allows the efficient optimization due to wide variability of shapes.

The automotive application of thin-walled rods with semi-opened cross section is studied in Chap. 11. The principal application of the theory of semi-opened thin-walled beams is the twist beam of the semisolid trail arm axle. The analytical expressions for the effective torsion stiffness and effective bending stiffness of the twist beam in terms of section properties of the twist beam with semi-opened cross section are derived. Based on the stiffness coefficients of the twist beam, the roll rate, chamber, and lateral rigidity of the suspension are derived.

This book is recommended primarily for engineers dealing with spring design and development, graduated from automotive or mechanical engineering courses in technical high school, or in other higher engineering schools. The researchers, working on elastic elements and energy harvesting equipment, will also find a general review for the fundamentals of spring technology.

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Contents

List of Symbols

Chapter 1

n	Number of active coils
L_0	Free length of the spring
$D = 2R$	Mean diameter of spring
$d = 2r$	Diameter of circular wire
G	Shear modulus
α	Pitch angle, lead angle
c	Compression (or extension) spring rate
$c_{\theta F}$	Compression-twist spring rate
c_{θ}	Twist spring rate
c^*	Design compression spring rate
W_T	Section modulus of torsion
τ	Basic (uncorrected) stress
τ_c	Corrected stress
$k = k(w)$	Correction factor
$w = D/d$	Spring index
$s = L_{rel} - L_{comp}$	Test spring travel
L_{rel}	Released length
L_{comp}	Compression length
U_e	Energy capacity of the linear spring
U_f	Work of applied forces
$F_{\min} \cdot F_{\max}$	Spring loads at lengths $L_{rel}\, L_{comp}$
V	Volume of the spring material
m	Mass of the spring material
$\tau_m = (\tau_{\min} + \tau_{\max})/2$	Mean stress in operation
$\tau_a = (\tau_{\max} - \tau_{\min})/2$	Stress amplitude in operation
τ_w	Working stress
τ_e	Endurance limit for completely reversed stress
S_f	Factor for safety

$p_{SWT\cdot\tau}$	Smith-Watson-Topper parameter for shear stress
$\gamma_a = \tau_a/G$	Shear strain amplitude
m_{opt}	Absolute lowest mass
d_{opt}	Optimal wire diameter
Q_p	Spring quality parameter
W_b	Bending section modulus of wire (for helical springs: with respect to helix axis)
W_{br}	Bending section modulus of wire (for helical springs: with respect radius of helix)
W_T	Twist section modulus of wire
EI	Bending stiffness of wire (for helical springs: about the helix axis)
I	Area moment of inertia of wire (for helical springs: about the helix axis)
EI_r	Bending stiffness of wire (for helical springs: with about radius of helix)
I_r	Area moment of inertia of wire (for helical springs: about the radius of helix)
GI_T	Torsional rigidity of wire about wire axis
I_T	Torsion constant for the section of wire
T	Height or thickness of the cross-section (for helical springs: in the direction of helix axis)
B	Width of the wire cross-section (for helical springs: in the radial direction)
A	Area of the wire cross-section
M_θ	Torque of the helical torsion spring
F_θ	Circumferential force in the wire direction
θ_F, θ_M	Reduction of the spring angle due to compression force and torque respectively
$\Delta D_F, \Delta D_M$,	Enlargement of the spring diameter due to compression force and torque
s_P, s_M	The spring travel due to compression force and torque
$\widetilde{U}_e$	Specific elastic energy density

Chapter 2

M_B	Bending couple
M_T	Twisting couple
α	Pitch angle
F	Axial force on the spring
$\psi(r, z)$	Warping function
$\bar{\xi}$, $\bar{k}$	Separation constants

$\varphi(r, z)$	Prandtl stress function
$Z_{+}(r),\ Z_{-}(r)$	Upper and lower sections of curve
c_w	Spring rate of one complete coil
$\phi_1(r)$, $\phi_2(z)$	Auxiliary functions
$R_i = D_i/2$	Inner radius of coil
$R_e = D_e/2$	Outer radius of coil
$T = Z_0\left(R_e^2 - R_i^2\right)$	Height of the wire cross-section
$B = R_e - R_i$	Width of the wire cross-section
T_i	Height of the inner opening in the hollow cross-section
B_i	Width of the inner opening in the hollow cross-section
$r_m = \sqrt{\left(R_e^2 + R_i^2\right)/2}$	Radius of maximum height of cross-section
V_1	Volume of a single coil
$z_{+}(r),\ z_{-}(r)$	Upper and lower contours of the inner curve
$\tau(\rho, \phi)$	Intensity of shear stress for the circular wire
$x = \rho \cos\phi$, $y = \rho \sin\phi$,	Cartesian coordinates of the circular cross-section
$0 \le \rho \le r$ $0 \le \phi \le 2\pi$	Polar coordinates of the circular cross-section
$w = D/d$	Index of spring with the circular wire
$\sigma_c(\phi) = \tau(\rho = r, \phi)$	Shear stress on the outer surface of the circular wire
σ_b	"Basic stress"
$\tau_{r\theta}^{(1)}, \tau_{z\theta}^{(1)}$	First degree Taylor polynomials of shear stresses
$\tau_{r\theta}^{(2)}, \tau_{z\theta}^{(2)}$	Second degree Taylor polynomials of shear stresses

Chapter 3

s	Axial displacement
s_Q	Lateral displacement
ϕ_Q	Angle of inclination of the bent axis
s_s	Displacement caused by the shear force
s_b	Displacement caused by bending moment
Q	Shear force
M_B	Bending moment
m_B	External torque per unit length
f_Q	External load in the transverse direction
$\langle EI_B \rangle$	Equivalent bending stiffness
$\langle GS \rangle$	Equivalent shear stiffness
A	Area of the wire cross-section
GI_T	Torsional rigidity of wire
d	Diameter of round wire
D	Mean diameter of coil

n_a	Number of active coils
ρ	Density of material
F	Static axial force
ω	Fundamental frequency
Λ	Inverse length parameter
β_i	Solutions of characteristic equation
ω_N	Circular natural frequencies of the spring
$\xi = L_o/D$	Slenderness ratio
$\mu = L/L_0$	Dimensionless relative length
L	Actual length of the spring under action of load
L_0	Free length of the spring
Ω_N	Relative fundamental frequency, $\Omega_N(\mu) = \omega_N/\omega_N^0$
ω_N^0	Natural frequency of free spring
$\mu_+^*(N), \mu_-^*(N)$	Critical deflection at loading and unloading

Chapter 4

s	Deviatoric stress
e	Deviatoric strain
$\sigma = Sp[\boldsymbol{\sigma}]$	Hydrostatic stress
$\varepsilon = Sp[\boldsymbol{\varepsilon}]$	Hydrostatic strain
T	Intensity of shear stress
Γ	Intensity of shear strain
G_p	Secant modulus
G_0	Shear modulus
ε_p	Plastic strain
σ_p	Plastic stress
k_s	Secant exponent
M_B	Bending moment in wire during plastic coiling
M_T	Torque of wire during plastic coiling
κ	Curvature in moment of plastic deformation
θ	Angle of twist per unit length in moment of plastic deformation (during plastic coiling)
$\rho = \sqrt{x^2 + y^2}$	Polar radius
$\varepsilon^* = r\kappa$	Maximal axial strain during plastic coiling
$\gamma^* = \theta r$	Maximal shear strain during plastic coiling
λ, μ	Dimensionless parameters
$\eta = \varepsilon_p/r$	Inverse length parameter
$\bar{R} = 1/\bar{\kappa}$	Bending radius after unloading (after spring-back)
$\bar{\theta}$	Angle of twist per unit length after spring-back
$\Delta\varepsilon$	Decrements of maximal axial strain

$\Delta\gamma$	Decrements of maximal shear strain
I	Second moment of inertia of circular wire
I_p	Polar moment of inertia of circular wire
$\overline{\sigma}_{zz}$, $\overline{\tau}_{\varphi z}$	Residual stresses after spring-back
$\chi = 2\pi\theta$	Instantaneous torsion of helix during plastic coiling
$\overline{\chi} = 2\pi\overline{\theta}$	Torsion of helix after spring-back
$\overline{\theta}$	Angle of twist per unit length after spring-back
$\bar{R}$	Unloaded radius after spring-back
$\bar{H}$	Unloaded pitch after spring-back
R	Coiling radius during plastic coiling
H	Coiling pitch during plastic coiling

Chapter 5

ϖ	Middle surface in the undeformed state
D_i	Inner diameter of middle surface of free spring
D_e	Outer diameter of middle surface of free spring
$r_i = D_i/2$	Inner radius of middle surface of free spring
$r_e = D_e/2$	Outer radius of middle surface of free spring
$\Delta = r_e/r_i$	Ratio of outer radius to inner radius
T	Material thickness of disk spring
$\mu = T/r_i$	Ratio of material thickness to inner radius
α	Slope angle of the undeformed middle surface
$x_e \leq x \leq x_i$	Coordinate on the meridian of the undeformed conical shell
c_i	Inversion center point for the cross-section
z_i, z_e	Heights of the inner and outer edges of the middle surface
h_z	Total height of the middle surface of the unloaded disk spring
h_r	Width of the middle surface of the unloaded disk spring
ψ	Slope angle of deformed middle surface
Ω	Middle surface of the shell in the deformed state
H_z	Height of middle surface in the deformed state Ω
H_r	Width of middle surface in the deformed state Ω
ε_1	Circumferential mid-surface strain:
κ_1	Circumferential curvature change
$\widetilde{h}_z$	Total height between utmost edges of the conical spring in its free state
$\widetilde{h}_r$	Width between utmost edges of the conical spring in its free state
$\widetilde{H}_z$	Total height between utmost edges of the deformed conical spring
$\widetilde{H}_r$	Total width of the deformed conical spring

s	Axial displacement, measured on the middle surface
$\widetilde{s}$	Axial displacement, measured from upper inside edge to lower outside edge
σ_1	Circumferential stress
σ_2	Meridional stress
σ_3	Stress normal to surface of the shell
E_1	Circumferential strain
E_2	Meridional strain
E_3	Strain normal to surface of the shell
E	Elasticity modulus (Young modulus)
ν	Poisson coefficient
G	Shear modulus
Π	Total potential energy
$U_e, U_1 \ldots U_5$	Elastic strain energy
U_f	Potential energy of the applied forces
M	Circumferential moment
F_z	Total axial force acting on the upper middle surface
F_R	Radial force acting on the upper middle surface
c_Z	Spring rate of the isotropic disk spring
$\widetilde{F}_{1Z}$	Corrected total axial
$\widetilde{c}_{1Z}$	Corrected spring rate of the isotropic disk spring
F_{AL}	Total axial force due to Almen and Laszlo
F_{zDIN}	Total axial force, DIN standard
c_{zDIN}	Spring rate, DIN standard
M_{AL}	Circumferential moment due to Almen and Laszlo
$\sigma_I, \sigma_{II}, \sigma_{III}, \sigma_{VI}$	Stresses on corner points of disk spring
σ_{Be}	Stress due to bending, outer diameter D_e
σ_{Bi}	Stress due to bending, inner diameter D_i.
σ_{Te}	Stress due to circumferential strain, outer diameter D_e
σ_{Ti}	Stress due to circumferential strain, inner diameter D_i.
N_1, N_2, N_{12}	Meridional, circumferential and shear direct forces
M_1, M_2, M_{12}	Meridional, circumferential and twist
C_{ij}	Membrane stiffness coefficients
D_{ij}	Flexural stiffness coefficients
Q_{ij}	Reduced stiffness coefficients
K_4	Effective circumferential elastic modulus
χ	Angle between meridian and principal material axis
q_{ij}	Reduced stiffness coefficients in local material system
E_1, E_2	Young's modules in two principal directions
ν_{12}, ν_{21}	Corresponding Poisson's ratios
$q_{66} = G_{12}$	Shear modulus
K_5	Effective circumferential elastic modulus for the orthotropic material
$K_{\max}$	Maximal value of effective circumferential elastic modulus

$K_{\min}$	Minimal value of effective circumferential elastic modulus
n_w	Number of waves of disk wave spring
$D_m = \dfrac{D_e + D_i}{2}$	Ring mean diameter of disk wave spring
$B = \dfrac{D_e - D_i}{2}$	Ring width of disk wave spring
$\lambda_w = a/l$	Length ratio of disk wave spring
l_w	Wave length of disk wave spring
c_w	Total initial spring rate of the wave spring
$\lvert\sigma_{Be} + \sigma_{Bi}\rvert/2$	Average bending stress
$(\sigma_{Te} - \sigma_{Ti})/2$	Average tensile stress
$\lvert\sigma_I\rvert, \lvert\sigma_{II}\rvert, \lvert\sigma_{III}\rvert, \lvert\sigma_{IV}\rvert$	Corner stresses
σ_R	"Comparative" maximal stress

Chapter 6

$f(\sigma_{eff}, t)$	Isotropic stress function
t	Time
$\dot{e}_{ij}$	Deviatoric component of creep strain
s_{ij}	Deviatoric component
σ_{eq}	Mises equivalent stress
c_τ	Creep constant for shear strain
c_σ	Creep constant for uniaxial strain
ξ	Stress exponent in Norton-Bailey and Naumenko-Altenbach-Gorash creep laws
ζ	Time exponent in Norton-Bailey and Garofalo laws
γ_e	Elastic component of shear strain
γ_c	Creep component of shear strain
M_T^0	Torque at the moment $t = 0$
$M_T(t)$	Torque as the function of time
$\Phi(t), \Psi(t)$	Relaxation function for coiled and disk springs
F_z^0	Spring force at the moment $t = 0$
$F_z(t)$	Spring force as the function of time
M_B^0	Bending moment at the moment $t = 0$
$M_B(t)$	Bending moment as the function of time
T	Thickness of disk spring
h	Free height of disk spring
ϕ	Rotation angle of the middle surface disk spring
$\alpha = h/(r_e - r_i)$	Initial cone angle of disk spring
C_i	Inversion point of disk spring during creep

Chapter 7

$f(\sigma_{eq}, t)$	Isotropic stress function
$D^{\hat{\alpha}} e_{ij}$	Deviatoric component of fractional creep strain
s_{ij}	Deviatoric component
σ_{eq}	Mises equivalent stress
c_τ	Creep constant for shear strain
c_σ	Creep constant for uniaxial strain
ξ	Stress exponent in an ordinary Norton-Bailey creep law
ζ	Time exponent in an ordinary Norton-Bailey creep law
γ_e	Elastic component of shear strain
γ_c	Creep component of shear strain
M_T^0	Torque at the moment $\mathbf{t} = 0$
$M_T(t)$	Torque as the function of time
Φ	Relaxation function
M_B^0	Bending moment at the moment $\mathbf{t} = 0$
$M_B(t)$	Bending moment as the function of time
$D^{\hat{\alpha}}$	Fractional derivative of order $0 < \hat{\alpha} < 1$
C_τ	Fractional creep constant for shear strain
C_σ	Fractional creep constant for uniaxial strain
$\hat{\alpha}$	An order of the Scott-Blair element
$\hat{\xi}$	Stress exponent in fractional Norton-Bailey creep law
$\hat{\zeta}$	Time exponent in fravtional Norton-Bailey creep law
$f_I(\sigma_{eq}, t)$	Isotropic stress function in the primary creep stage
$f_{II}(\sigma_{eq}, t)$	Isotropic stress function in the secondary creep stage
$f\left(\sigma_{eq}, t\right) = h(t)s\left(\sigma_{eq}\right)$	Decomposition of an isotropic stress function
$\tau_{\max}$	Shear stress on the surface of circular wire
$\sigma_{\max}$	Normal stress at the uppermost edge of the beam.

Chapter 8

$K_{\max}$	Maximum stress intensity factor per cycle
$K_{\min}$	Minimum stress intensity factor per cycle
K	Range of stress intensity factor
$\sigma = \sigma_{\max} - \sigma_{\min}$	Stress range, $\sigma = 2\sigma_a$
$\sigma_{\max}$	Maximum stress per cycle, $\sigma_{\max} = \sigma_m + \sigma_a$
$\sigma_{\min}$	Minimum stress per cycle, $\sigma_{\min} = \sigma_m - \sigma_a$
Y	Dimensionless geometry parameter
$\sigma_f^{'}$	Fatigue strength coefficient for normal stress
$\tau_f^{'}$	Fatigue strength coefficient for shear stress

b_σ	Fatigue strength exponent.
$c_f(R_\sigma)$	Material constant for a given stress ratio R_σ
R_σ	Stress ratio of cyclic load
$K_m = (K_{\max} + K_{\min})/2$	Mean value of stress intensity factor
$U_I(K)$	Unified propagation function of type I
$U_{II}(K)$	Unified propagation function of type II
$p > 1$	Fatigue exponent
$m_2 > 1$	Exponent at short-term limit
$m_1 > 1$	Endurance limit exponent
K_2	Short-term threshold limit
K_1	Endurance threshold limit
$n_f(a, \delta, \sigma)$	Number of cycles to failure as function of stress range
$U_{I,i}(K), i = 1, 2, 3$	Limit cases for propagation function of type I
$U_{II,i}(K), i = 1, 2, 3$	Limit cases for propagation function of type II
$n_{I,i}(a, \sigma), i = 1, 2, 3$	Limit cases for number of cycles to fault (type I)
$n_{II,i}(a, \sigma), i = 1, 2, 3$	Limit cases for number of cycles to fault (type II)
$N_f(\sigma_a, \sigma_m)$	Number of cycles to failure as function of stress amplitude and mean stress
$\Lambda\left[R_\sigma, {R_\sigma}^*\right] = \lambda_r[R_\sigma]/\lambda_r\left[{R_\sigma}^*\right]$	Relation between fatigue coefficients

Chapter 9

Ω	Volume of the whole structural element
V_0	Average volume that contains one critical defect.
A_0	Average surface element that contains one critical defect
P_S	Survival probability
$P_F = 1 - P_S$	Failure probability
$g(\sigma)$	Weibull immediate "risk of rupture"
$h(N, \sigma)$	Weibull "risk of rupture" of the element after N cycles
S_{sh}, S_W, m_W	Weibull stress shift, stress scale and shape parameters
$\lambda_f \equiv \frac{1}{2}\left(\sigma_f'\right)^{-1/b_\sigma}$	Constant in fatigue equation
$b_\sigma = -1/p$	Strength exponent
$p = -1/b_\sigma$	Reciprocal strength exponent
σ_0, N_0	Auxiliary scaling constants, $\sigma_f' = \sigma_0(2N_0)^{-b_s}$
N_F	Lowest cycles number to the failure for a given stress amplitude (failure event of the first homogeneously stressed specimen)
N_L	Highest cycles number to the failure for a given stress amplitude (failure event of the last homogeneously stressed specimen)
k_f	Width constant of failure region
$d = 2r$	Wire diameter and radius

$D = 2R$	Mean coil diameter and radius
L	Length of wire
$A_e = 2\pi rL$	Outer surface of the whole structural element, outer surface of the spring wire
$V = \pi r^2 L$	Volume of the wire material of the helical spring
T, B	Axes of the elliptic cross-section of wire
$\tau_{r\theta}, \tau_{z\theta}$	Shear stress components in the cross-section of wire
θ	Torsion angle pro length unit
G	Shear modulus
$P_{S.i}$	Survival probability of the spring with defects
$P_{R.i}$	Survival probability of the straight rod with defects
k_1, k_2	Auxiliary functions for survival probabilities and ratio of cycles to failure of spring to straight rod
α_i	Ratio of survival probabilities of spring and rod
N_S	Number of cycles to failure of helical spring
P_S^*	Prescribed survival probability of the structural element, helical spring
N_R	Number of cycles to failure of straight rod

Chapter 10

I_{xc}	Moment of inertia of the cross-section with respect to the x-axis
I_{yc}	Moment of inertia of the cross-section with respect to the y-axis
x_c	The x-coordinate of the center of mass of the cross-section
y_c	The y-coordinate of the center of mass of the cross-section
$S_{\omega x}$	Static moment of the cross-section with respect to the x-axis
$S_{\omega y}$	Static moment of the cross-section with respect to the y-axis
I_ω	Sectorial moment of inertia of the section
α_x	The x-coordinate of the twist centre of the cross-section
α_y	The y-coordinate of the twist centre of the cross-section
I_d	Geometrical torque stiffness of inertia of the section
A	The area of the material part of the cross-section
A_m	The area enclosed by the curve L_m

Chapter 11

r_a	Axle roll stiffness, $r_a = i^2 r_t$,
r_t	Torsion stiffness of the twist beam
$i_t = L/L_T$	Geometrical transmission ratio

L_T	Length of trailing arm
L	Length of twist beam length
r_l	Lateral stiffness of the axle
r_c	Camber stiffness of the axle
r_z	Bending stiffness of the twist beam
$\lambda_c{}^2 = \frac{GI_T}{EI_\omega}$	Characteristic length
$\bar{r}_t$	Torsion stiffness of the twisted rod without the Influence of bi-moment
$K(\lambda_c L)$	Stiffening factor due to bi-moment
B_m	Bi-moment
M_H	Moment due to pure torsion
M_S	Moment due to constrained torsion
$\sigma_z^{(i)}$	Normal stress due to torsion
$F_\tau(z, s)$	Flow of shear stress
τ_S	Shear stress due to bi-moment
τ_H	Shear stress due to pure torsion
τ	Total shear stress
σ_z	Normal stress due to bending
σ_v	Equivalent tensile stress