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Giuseppe De Nittis · Max Lein

Linear Response Theory

An Analytic-Algebraic Approach

 Springer

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ISSN 2197-1757 ISSN 2197-1765 (electronic)
SpringerBriefs in Mathematical Physics
ISBN 978-3-319-56731-0 ISBN 978-3-319-56732-7 (eBook)
DOI 10.1007/978-3-319-56732-7

Library of Congress Control Number: 2017937511

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Printed on acid-free paper

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The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

For Raffa & Tenko

Acknowledgements

We are indebted to François Germinet who introduced G. D. to [7, 27], the works which have inspired this whole endeavor. In addition to François, many other colleagues were not just kind enough to share their insights with us, but also encouraged us to see this book project to an end. We would particularly like to express our gratitude to Jean Bellissard, Hermann Schulz-Baldes, Daniel Lenz, Georgi Raikov, and Massimo Moscolari. Thanks to their help, we were able to overcome many of the obstacles more quickly and more elegantly.

Lastly, G. D. thanks FONDECYT which supported this book project through the grant “*Iniciación en Investigación 2015—N° 11150143*”.

Santiago, Chile
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