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Mathematical Methods of Classical Physics

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Contents

Symbols

A	Connection one-form, 69
$\mathscr{A}_G^{YM}(E)$	Subset of Yang–Mills G-connections, 69
$\mathrm{Aut}_G(E)$	Gauge group of the vector bundle E with G-reduction $\mathscr{F}_G(E)$, 68
$\mathbf{B}$	Magnetic field, 65
$\mathrm{Diff}^+(M)$	Subgroup of orientation preserving diffeomorphisms of M, 53
$\mathrm{Div}\,P$	Total divergence, 51
d^∇	Covariant exterior derivative, 65
$dvol$	Volume element on $\mathfrak{S}$, 48
E	(Total) energy, 6
E_a	Component of Euler–Lagrange operator, 50
E_{kin}	Kinetic energy, 5
E_{pot}	Potential energy, 6
$\mathbf{E}$	Electric field, 65
$\mathscr{F}(\mathrm{E})$	Frame bundle of E, 64
F^∇	Curvature of a connection ∇, 64
$\tilde{f}$	Legendre transform of a smooth function f, 28
g	Metric on M (Riemannian or pseudo-Riemannian), 6
H	Hamiltonian, 21
$\mathrm{Isom}^+(N,h)$	Subgroup of orientation preserving isometries of the metric h on N, 63
J	(Noether) current, 60
J^0	Charge density, 60
$\mathrm{Jet}^k(\mathfrak{S},\mathscr{T})$	Jet bundle over $\mathfrak{S}$, 48
$j^k(f)$	Smooth section of the jet bundle $\mathrm{Jet}^k(\mathfrak{S},\mathscr{T})$, 48
$\mathbf{J}$	Flux density, 60
$\mathbb{K}$	Field (in this book, $\mathbb{K}=\mathbb{R}$ or $\mathbb{C}$), 64
L	Length of the angular momentum vector $\mathbf{L}$, 14
$\mathscr{L}$	Lagrangian (function), 5

$\mathscr{L}_2$	Linearized Lagrangian, 31
$\mathbf{L}$	Angular momentum vector, 14
M	Smooth manifold (configuration space), 5
m	Mass of a point particle, 6
n	Dimension of M, 8
$\mathrm{pr}^{(k)}Z$	k-th prolongation of Z, 54
$\mathbf{p}$	Momentum vector, 13
Q	(Noether) charge, 60
$(q^1,\ldots,q^n,\hat{q}^1,\ldots,\hat{q}^n)$	Induced/canonical local coordinates on *TM* associated with local coordinates $(x^1,\ldots,x^n)$ on M, 7
$(q^1,\ldots,q^n,p_1,\ldots,p_n)$	Darboux coordinates, 20
Ric	Ricci curvature tensor, 72
S	Action, 6
$scal$	Scalar curvature of (M, g), 72
$S[f]$	Action functional of a classical field theory, 47
$\mathfrak{S}$	Source manifold (possibly with boundary), 45
t	Time, 5
$\mathscr{T}$	Target manifold, 47
TM	Tangent bundle of M, 5
T^{μ}_{ν}	Component of the energy–momentum tensor, 76
V	Potential, 5
V_{eff}	Effective potential, 15
$(x^1,\ldots,x^n)$	Local coordinates on an open subset $U \subset M$, 7
X_f	Hamiltonian vector field associated with a smooth function f, 20
$\mathfrak{X}(M)$	Set of all smooth vector fields on M, 11
X^{ver}	Vertical lift of X, 11
α	Euler–Lagrange one-form, 50
α_i	Component of the Euler–Lagrange one-form in some local coordinate system, 8
δ_{ij}	Kronecker delta. Its value is defined to be 1 if the indices are equal, and 0 otherwise, 33
γ	Smooth curve in M, 6
κ	Gravitational coupling constant, 72
λ	Liouville form, 20
Λ	Cosmological constant, 75
Ω	Hessian matrix of the potential V, 32
ω	Canonical symplectic form, 20
ω_0	Frequency of a small oscillation, 33
π	Canonical projection from *TM* to M, 7
$\tau(f)$	Tension of f, 62
τ_0	Period of a small oscillation, 33
ξ	Eigenvector of W, 35
$\langle\cdot,\cdot\rangle$	Euclidean scalar product on $\mathbb{R}^n$, 5

$*$	Hodge star operator, 65
Δ	Laplace operator, 63
$\dot{(\cdot)}$	Time derivate, $\dot{f}(t) = f'(t), \ddot{f}(t) = f''(t)$, 2
∇	Covariant derivative or connection, 9
Σ	We use an adapted version of *Einstein's summation convention* throughout this book: Upper and lower indices appearing with the same symbol within a term are to be summed over. We write the symbol Σ to indicate whenever this convention is employed. Owing to the aforementioned convention, the summation indices can be (and are usually) omitted below the symbol Σ, 7
$(\cdot)^{\mathsf{T}}$	Matrix transposition, 34