

Advanced Structured Materials

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Holm Altenbach · Robert V. Goldstein
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Editors

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Editors

Holm Altenbach
Institut für Mechanik
Otto-von-Guericke-Universität
Magdeburg
Germany

Robert V. Goldstein
Mechanics of Strength and Fracture of
Materials and Structures
Ishlinsky Institute for Problems in
Mechanics of the Russian Academy of
Sciences
Moscow
Russia

Evgenii Murashkin
Modelling in Solid Mechanics
Ishlinsky Institute for Problems in
Mechanics of the Russian Academy of
Sciences
Moscow
Russia

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Preface



Professor Alexander V. Manzhurov is one of the leading Russian researchers in solid mechanics and applied mathematics, a world-wide authority who has exerted a major influence on research in the areas of *Mechanics of Growing Solids*, *Mechanical Analysis of Additive Manufacturing*, *Contact Mechanics and Tribology*, *Viscoelasticity*, *Creep* and *Integral Equations and their Applications*. This volume of the Advanced Structured Materials Series is dedicated to his sixtieth birthday and contains a selection of research papers written by friends and colleagues from various countries, including China, Germany, India, Russia, South Africa, the UK, and the USA. Some of these papers deal with re-

search fields close to Prof. Manzhurov's scientific activities, while the other ones show new trends in solid mechanics which could be combined with his scientific interests in the future.

Professor Manzhurov was born on 24 May 1957 in Rostov-on-Don, Russia. His scientific carrier is related to different places:

- In 1979 he obtained his diploma (equivalent to M.Sc. degree) in Mechanics (Diploma with Honors) at the Rostov State University, Department of Mechanics and Mathematics, where he studied from 1974 - 1979
- In 1982 he was employed at the Institute for Problems in Mechanics of the Academy of Sciences of the USSR (at present the Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences)
- In 1983 he defends his doctoral thesis (candidate of physico-mathematical sciences) at the Moscow Institute of Electronic Engineering Industry (he was a PhD student at the Kuibyshev Moscow Institute of Civil Engineering from 1979 to 1982)

- In 1993 he defends his second doctoral thesis (doctor of physico-mathematical sciences, equivalent to the DSc degree or doctor habilitatus) at the Institute for Problems in Mechanics of the Russian Academy of Sciences) which was accepted by the Higher Attestation Commission of the Russian Federation
- Since 2014 he is Foreign Member of the National Academy of Sciences of the Republic of Armenia

In 1998 he obtained the certificate of professor in mathematics by the Higher Certification Committee of the Russian Federation.

At the moment he serves as Deputy Director and Head of Department for Modeling in Solid Mechanics at the Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, as Head of Branch Department of Applied Mathematics at the Bauman Moscow State Technical University, as Professor of Department of Higher Mathematics at the National Research Nuclear University (Moscow Engineering Physics Institute) and as Professor of Department of Higher Mathematics at the Moscow Technological University.

At the same time he is Vice Chairman of the Solid Mechanics Scientific Council of the Russian Academy of Sciences, a member of the Mathematics and Mechanics Expert Council of the Higher Attestation Commission of the Russian Federation, and a member of the Mathematics, Mechanics and Computer Science Expert Council of the Russian Foundation for Basic Research. In addition, he is a member of:

- Russian National Committee on Theoretical and Applied Mechanics (RNCTAM)
- European Mechanics Society (EUROMECH)
- American Society of Mechanical Engineers (ASME)
- American Mathematical Society (AMS)
- Gesellschaft für Angewandte Mathematik und Mechanik (GAMM)
- International Association of Engineers (IAENG)
- Research Board of Advisors of the American Biographical Institute (ABI)
- Editorial Boards of about a dozen scientific journals, including Mechanics of Solids (MTT), Proceedings of National Academy of Sciences of Armenia - Mechanics, Computational Mechanics of Continua and of the scientific website The World of Mathematical Equations (EqWorld)

His main awards

- Russian State Scientific Scholarship for Outstanding Scientists 1997, 2003
- First Competition of the Science Support Foundation, 2001
- Certificate of Honor of the Russian Foundation for Basic Research, 2016
- Certificate of Honor of the Higher Certification Committee of the Ministry of Education and Science of the Russian Federation, 2016

He is an author or co-author of 19 books and textbooks in English, German and Russian including some best-selling handbooks. In addition, he published more than 200 research papers and obtained two inventor's certificates. Selected publications can be find at the end of the preface.

All of these publications, scientific achievements and other contributions have given us great respect for him, and we regard him as a perfect scientist. At the same time,

he is a man of humor. He is a husband, father of three beautiful girls, loves travelling and likes to get new and unexpected impressions. We are grateful to Prof. Manzhirov for his scientific contributions to solid mechanics and applied mathematics, and believe that he will continue to have an impact in research.

We have to thank all contributors for their perfect job. Last but not least, we gratefully acknowledge Dr. Christoph Baumann (Springer Publisher) for support of the book project.

Magdeburg, Moscow,
May 2017

*Holm Altenbach
Robert Goldstein
Evgenii Murashkin*

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List of Contributors

Leonid D. Akulenko

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: kumak@ipmnet.ru

Sergei Alexandrov

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: sergei_alexandrov@yahoo.com

Holm Altenbach

Institut für Mechanik, Otto-von-Guericke-Universität Magdeburg, Universitätsplatz
2, 39106 Magdeburg, Germany, e-mail: holm.altenbach@ovgu.de

Boris D. Annin

Lavrentyev Institute of Hydrodynamics of Siberian Branch of the RAS,
Lavrentyev Ave. 15 630090 Novosibirsk, Russia, e-mail: annin@hydro.nsc.ru

Marcus Aßmus

Institut für Mechanik, Otto-von-Guericke-Universität Magdeburg, Universitätsplatz
2, 39106 Magdeburg, Germany, e-mail: marcus.assmus@ovgu.de

Olga M. Babeshko

Kuban State University, Stavropolckia st. 149, 350040 Krasnodar, Russia, e-mail:
babeshko49@mail.ru

Vladimir A. Babeshko

Southern Scientific Center RAS, Chehov st. 41, 344006 Rostov-on-Don &
Kuban State University, Stavropolckia st. 149, 350040 Krasnodar, Russia, e-mail:
babeshko41@mail.ru

Alexandra S. Begun

Institute of automation and control processes of FEB RAS, 690041 Vladivostok,
Russia, e-mail: asustinova@dvo.ru

Stefan Bergmann

Institut für Mechanik, Otto-von-Guericke-Universität Magdeburg, Universitätsplatz 2, 39106 Magdeburg, Germany, e-mail: stefan.bergmann@st.ovgu.de

Judith Nomantande Maureen Bidie

Department of Mathematics and Statistics, Tshwane University of Technology, South Africa, e-mail: bidiejnm@tut.ac.za

Alexander A. Burenin

Institute of Machine Science and metallurgy of FEB RAS, 681005 Komsomol'sk-on-Amur, Russia, e-mail: burenin@dvo.ru

Johanna Eisenträger

Institut für Mechanik, Otto-von-Guericke-Universität Magdeburg, Universitätsplatz 2, 39106 Magdeburg, Germany, e-mail: johanna.eisentraeger@ovgu.de

Victor A. Eremeyev

Southern Federal University, Institute of Mathematics, Mechanics and Computer Science, Milchakova Street 8a, Rostov-on-Don, 344090, Russia and Rzeszów University of Technology, Faculty of Mechanical Engineering and Avionics, al. Powstańców Warszawy 8, 35959 Rzeszów, Poland, e-mail: eremeyev.victor@gmail.com; veremeyev@prz.edu.pl

Olga V. Evdokimova

Southern Scientific Center RAS, Chehov st. 41, 344006 Rostov-on-Don, Russia, e-mail: evdokimova.olga@mail.ru

Igor A. Fedotov

Department of Mathematics and Statistics, Tshwane University of Technology, South Africa, e-mail: FedotovI@tut.ac.za

Gennady V. Garkushin

Joint Institute for High Temperatures of the Russian Academy of Sciences, Izhorskaya st. 13 bldg 2, 125412 Moscow, Russia

Dimitri V. Georgievskii

Lomonosov Moscow State University, Leninskie Gory 1, 119991 Moscow, Russia, e-mail: georgiev@mech.math.msu.su

Robert V. Goldstein

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1, 119526 Moscow, Russia, e-mail: goldst@ipmnet.ru

Kondagunta Veeraraghava Nagendra Gopal

Department of Aerospace Engineering, Indian Institute of Technology, Madras, India, e-mail: gopal@ae.iitm.ac.in

Valentin A. Gorodtsov

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1, 119526 Moscow, Russia, e-mail: gorod@ipmnet.ru

Irina G. Goryacheva

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: goryache@ipmnet.ru

Narinder Kumar Gupta

Department of Applied Mechanics, Indian Institute of Technology, Delhi, India,
e-mail: narinder_gupta@yahoo.com

Dmitry A. Indeitsev

Institute for Problems in Mechanical Engineering RAS, V.O., Bolshoy pr., 61,
199178 St. Petersburg, Russia, e-mail: dmitry.indeitsev@gmail.com

Elena Ivanova

Peter the Great Saint-Petersburg Polytechnic University, Politekhnikeskaja 29,
195251 St.-Petersburg, Russia, e-mail: elenaivanova239@gmail.com

Stephan V. Joubert

Department of Mathematics and Statistics, Tshwane Univesity of Technology,
South Africa, e-mail: JoubertSV@tut.ac.za

Gennady I. Kanel

Joint Institute for High Temperatures of the Russian Academy of Sciences,
Izhorskaya st. 13 bldg 2, 125412 Moscow, Russia, e-mail: kanel@ficp.ac.ru

Julius Kaplunov

Keele University, Great Britain, e-mail: j.kaplunov@keele.ac.uk

Alexander V. Kaptsov

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: kaptsov@ipmnet.ru

Vladimir I. Karev

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: wikarev@ipmnet.ru

Kirill E. Kazakov

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow & Bauman Moscow State Technical University, Baumanskaya
st. 5 bldg 1, 105005 Moscow, Russia, e-mail: kazakov-ke@yandex.ru

Dmitry M. Klimov

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia

Alexander G. Kolpakov

Siberian State University of Telecommunications and Informatics, Kirov st. 86,
630009 Novosibirsk, Russia, e-mail: algk@ngs.ru

Yury F. Kovalenko

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: perfolinkgeo@yandex.ru

Larisa V. Kovtanyuk

Institute of automation and control processes of FEB RAS, Vladivostok Russia,
690041, e-mail: lk@dvo.ru

Sergey A. Kumakshev

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: kumak@ipmnet.ru

Ilya I. Kudish

Department of Mathematics, Kettering University, Flint, MI, 48504, USA, e-mail:
ikudish@kettering.edu

Svetlana P. Kurdina

Bauman Moscow State Technical University, Baumanskaya st. 5 bldg 1, 105005
Moscow, Russia, e-mail: svetlana-ka@yandex.ru

Masodi Lesiba Glen Lekalakala

Department of Mathematics, Tshwane University of Technology, South Africa,
e-mail: glen@vut.ac.za

Sergei A. Lychev

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: lychevsa@mail.ru

Yulia A. Mochalova

Institute for Problems in Mechanical Engineering RAS, V.O., Bolshoy pr., 61,
199178 St. Petersburg, Russia, e-mail: yumochalova@yandex.ru

Nikita F. Morozov

St. Petersburg State University, Universitetskaya nab. 7/9, 199034 St.Petersburg,
Russia, e-mail: morozov@nm1016.spb.edu

Wolfgang H. Müller

Institute of Mechanics, Chair of Continuum Mechanics and Materials Theory,
Technical University of Berlin, Einsteinufer 5, 10587 Berlin, Germany e-mail:
whmueller1000@gmail.com

Evegnii V. Murashkin

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Bauman Moscow State Technical University, Baumanskaya st.
5 bldg 1, 105005 Moscow, National Research Nuclear University "MEPhI" (Moscow
Engineering Physics Institute), Kashirskoye shosse 31, 115409 Moscow, & Moscow
Technological University, Vernadsky Ave 78, 119454 Moscow, Russia, e-mail:
murashkin@ipmnet.ru, evmurashkin@gmail.com

Konstantin Naumenko

Institut für Mechanik, Otto-von-Guericke-Universität Magdeburg, Universitätsplatz
2, 39106 Magdeburg, Germany, e-mail: konstantin.naumenko@ovgu.de

Galina L. Panchenko,
Institute of Machine Science and metallurgy of FEB RAS, 681005 Komsomolsk-
on-Amur, Russia

Dmitry A. Parshin
Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow & Bauman Moscow State Technical University, Baumanskaya st.
5 bldg 1, 105005 Moscow, Russia, e-mail: parshin@ipmnet.ru

Alexander V. Pavlenko
Joint Institute for High Temperatures of the Russian Academy of Sciences,
Izhorskaya st. 13 bldg 2, 125412 Moscow, Russia

Danila A. Prikazchikov
School of Computing and Mathematics, Keele University, Keele, Staffordshire,
ST5 5BG, UK, e-mail: d.prikazchikov@keele.ac.uk

Alexander N. Prokudin
Institute of Machine Science and metallurgy of FEB RAS, 681005 Komsomolsk-
on-Amur, Russia, e-mail: sunbeam_85@mail.ru

Yuri N. Radayev
Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: radayev@ipmnet.ru, y.radayev@gmail.
com

Sergei I. Rakin
Siberian State Transport University, Dusi Koval'chuk st. 191, 630049 Novosibirsk,
Russia,
e-mail: rakinsi@ngs.ru

Abishera Ravichandra Rajkumar
Department of Aerospace Engineering, Indian Institute of Technology, Madras,
India

Sergey V. Razorenov
Joint Institute for High Temperatures of the Russian Academy of Sciences,
Izhorskaya st. 13 bldg 2, 125412 Moscow, Russia

Andrey S. Savinykh
Joint Institute for High Temperatures of the Russian Academy of Sciences,
Izhorskaya st. 13 bldg 2, 125412 Moscow, Russia

Olga Sergushova
School of Computing and Mathematics, Keele University, Keele, Staffordshire,
ST5 5BG, UK, e-mail: o.sergushova@keele.ac.uk

Michael Y. Shatalov
Department of Mathematics and Statistics, Tshwane University of Technology,
South Africa, e-mail: ShatalovM@tut.ac.za

Efim I. Shifrin

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: shifrin@ipmnet.ru

Hiltje Spoelstra

Department of Mathematics and Statistics, Tshwane Univesity of Technology,
South Africa, e-mail: SpoelstraH@tut.ac.za

Feodor I. Stepanov

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: stepanov_ipm@gmail.ru

Elena V. Torskaya

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: torskaya@ipmnet.ru

Petr E. Tovstik

St. Petersburg State University, Universitetskaya nab. 7/9, 199034 St.Petersburg,
Russia, e-mail: peter.tovstik@mail.ru

Tatyana P. Tovstik

Institute of Problems of Mechanical Engineering RAS, Bol'shoi pr. 61, St.
Petersburg, 199178, Russia, e-mail: tovstik_t@mail.ru

Konstantin B. Ustinov

Ishlinsky Institute for Problems in Mechanics RAS, Vernadskiy Ave, 101 bldg 1,
119526 Moscow, Russia, e-mail: ustinov@ipmnet.ru

Ramachandran Velmurugan

Department of Aerospace Engineering, Indian Institute of Technology, Madras,
India, e-mail: ramanv@iitm.ac.in

Elena Vilchevskaya

Institute for Problems in Mechanical Engineering of the Russian Academy of
Sciences, Bol'shoi pr. 61, V.O., 199178 St. Petersburg, Russia & Peter the
Great Saint-Petersburg Polytechnic University, Politekhnicheskaja 29, 195251
St.-Petersburg, Russia, e-mail: vilchevska@gmail.com

Si-Yu Wang

Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University,
Yanchang Road 149, 200072 Shanghai, China, e-mail: 343396676@qq.com

Heng Xiao

Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University,
Yanchang Road 149, 200072 Shanghai, China, e-mail: xiaoheng@shu.edu.cn

Zheng-Nan Yin

Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University,
Yanchang Road 149, 200072 Shanghai, China, e-mail: znyin@shu.edu.cn

Lin Zhan

Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University,
Yanchang Road 149, 200072 Shanghai, China, e-mail: 302974155@qq.com