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Kenneth R. Meyer • Daniel C. Offin

Introduction to Hamiltonian Dynamical Systems and the N-Body Problem

Third Edition

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Preface

The theory of Hamiltonian systems is a vast subject which can be studied from many different viewpoints. This book develops the basic theory of Hamiltonian differential equations from a dynamical systems point of view.

In this third edition, the first four chapters give a sound grounding in Hamiltonian theory and celestial mechanics making it suitable for an advanced undergraduate or beginning graduate course. It contains an expanded treatment of the restricted three-body problem including a new derivation, a treatment of Hill's regions, discrete symmetries, and more. It also has a detailed presentation of the symmetries, the moment map, Noether's theorem, and the Meyer-Marsden-Weinstein reduction theorem with applications to the three-body problem. Also included is an introduction to singular reduction and orbifolds with application to bifurcation of periodic solutions along with an introduction of the lemniscate functions used for bounded and stability results.

This edition retains and advances the treatment of such advanced topics as parametric stability, conical forms for Hamiltonian matrices, symplectic geometry, Maslov index, bifurcation theory, variational methods, and stability results including KAM theory.

It assumes a basic knowledge of linear algebra, advanced calculus, and differential equations but does not assume the advanced topics such as Lebesgue integration, Banach spaces, or Lie algebras. Some theorems which require long technical proofs are stated without proof but only on rare occasions.

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Invitation

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