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Juan C. Vallejo • Miguel A.F. Sanjuan

Predictability of Chaotic Dynamics

A Finite-time Lyapunov Exponents Approach

Springer

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To my wife Laura and my children Alicia and Pedro

To my wife Céline and my daughters Alicia and Mónica

Preface

Mankind has always been concerned with the desire of understanding the universe, knowing the ultimate reasons behind past events and having the ability of forecasting the future ones. In a very simplified view, the main task of a physicist is to observe the nature, to build models and to derive predictions from them. But, in some fields, as, for instance, astronomy, one recollects the necessary information from observed objects without the possibility of having direct access to them, so one has not the possibility of altering the key parameters of the studied objects. Even worst, the timescales applicable may be out of the human timescales. A key issue in these cases is to study the subject of observation through numerical simulations.

In recent years, simulations have gained much relevance in physics. They play an important role as a new tool in addition to theory and experiments. Furthermore, they can be very useful in exploring the consequences of varying the parameters in physical models. With the widespread usage of computer simulations to solve complex dynamical systems, the reliability of the numerical calculations is of increasing interest. We can take a model, or set of equations describing the system, and integrate it during a certain time interval. The question to answer here is: how valid is the resulting forecast? Every model has inherent inaccuracies leading its results to deviate from the true solution. Any numerical schema used for solving it will introduce several errors. Round-off errors are present because it is impossible to represent all real numbers exactly on a machine with finite memory. Truncation errors are committed when the iterative method is terminated or a mathematical procedure is approximated and the approximate solution differs from the exact solution. Or discretisation errors must be taken into account when the solution of the discrete problem does not coincide with the solution of the continuous problem.

The reliability of the calculations is directly related to the regularity and instability properties of the modelled flow. This is an interdisciplinary scenario, where the underlying physics provides the simulated models, nonlinear dynamics provides their chaoticity and instability properties and the computer sciences provide the actual numerical implementation.

This book faces the problem of characterising the time (predictability time) when a numerical prediction can be considered valid by using techniques and concepts derived from the nonlinear dynamics and chaos theory.

Because of the possible differences between the real problem and the model used for making predictions, and because the numerical methods will introduce different errors and perturbations, the resulting numerical solution of a model will not match with the real one.

A system is said to be chaotic when it presents strong sensitivity to initial conditions. It is obvious that the presence of chaos can impose certain limits to the time when two initially closed trajectories, the real one and the calculated one, will remain close. Chaos does not always imply a low predictability. An orbit can be chaotic and still be predictable, in the sense that the chaotic orbit is followed, or shadowed, by a real orbit, thus making its predictions physically valid. The computed orbit may lead to right predictions despite being chaotic because of the existence of a nearby exact solution. This true orbit is called a shadow, and the existence of shadow orbits is a very strong property.

There are several books that deal with the selection of the most suitable numerical scheme for solving a given problem, others that describe different chaos indicators for characterising the presence of chaos and others that perform a thoroughly theoretical study of the underlying shadowing theories.

This book aims to take a different approach and it performs a descriptive analysis of how one can gain insight in the study of the predictability of a system. This characterisation will be done through the computation of the finite-time Lyapunov exponents and their distributions. This book will circumscribe to the field of the dynamical continuum flows, even when maps and discrete systems are mentioned when needed as part of the discussion.

As a consequence, this book presents basic concepts on dynamical flows needed for the computation of asymptotic and finite-time Lyapunov exponents. It will show how this computation can provide different properties of a given system related to its predictability.

This book approaches these issues from a numerical exploration perspective. The mathematical background is not detailed, but just introduced and appropriate bibliographic references are given to the interested reader. It does not focus on the mathematical derivations of required well-known algorithms and schemes. Conversely, it describes how to use them for the purpose of this work, that is, analysing the predictability of a given system. So, it focus, on describing the procedures required for carrying out these analyses in a step-by-step method. After discussing these procedures, we present in each chapter a reduced set of simple case studies on conservative and dissipative systems.

This book is primarily developed as a text at the postgraduate level and also as a reference book for researchers working and/or interested in the field of the predictability of dynamical systems. It is a self-contained book, where all needed techniques are properly described so that a reader is able to reproduce the results presented and may apply them to any problem of his/her interest.

This work is structured into four main chapters and one appendix. This monograph has been written in a way that every chapter can be read independently of the others. This self-consistency means that some overlap may be found when reading the whole book, but this overlapping will allow the interested reader to directly consult some sections and methods described in a given section of the book.

The book begins with an introduction to the ideas of forecasting in science in order to give a historical perspective. This introductory part discusses the forecasting and its relationship with the predictability of the numerical computations. The second chapter is devoted to the calculus of the finite-time Lyapunov exponents and describes how their distributions provide information of the properties of a dynamical flow at local scales. The third chapter expands the previous ideas, and presents how the distributions of finite-time Lyapunov exponents change as the different regimes of the dynamical flow are reached, and how these changes can be used for characterising the system. The final chapter is concerned with the calculus of the predictability itself, in terms of presence of the shadowing property, as indicator of the reliability of any results obtained from solving numerically a given dynamical system. The appendix describes the main algorithms used along the book for computing the finite-time Lyapunov exponents, with some comments about how to implement and use them in the most efficient way.

We are indebted to Ricardo L. Viana, Juergen Kurths and James A. Yorke for their fruitful comments and discussions concerning the computation of finite-time Lyapunov exponents at very small local scales. Finally, we acknowledge the support and encouragement given by our family members during the preparation of this monograph.

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November 2016

Contents

Acronyms

We have tried to reduce the number of abbreviations used along the text. However, there is a variety of notations and acronyms found in the related literature and here we summarise them, as well as the most frequently used symbols found along the text.

AFTLE	Averaged Lyapunov exponent
DLE	Direct Lyapunov exponent
ELN	Effective Lyapunov number
FTLE	Finite-Time Lyapunov exponent
FSLE	Finite-Size Lyapunov exponent
LN	Lyapunov number
LCE	Lyapunov characteristic exponent
LCI	Lyapunov characteristic indicator
LCN	Lyapunov Characteristic number
MLE	Maximal Lyapunov exponent
ODE	Ordinary Differential equation
SDLE	Scale-dependant Lyapunov exponent
SVD	Singular value decomposition
UDV	Unstable dimension variability
UPO	Unstable periodic orbit

χ	Finite-time or short-time Lyapunov exponent, FTLE
Δt	Finite-time interval length
$\boldsymbol{\phi}(\boldsymbol{x}, t)$	Solution of the flow equation
P_+	Probability of positivity
h	Predictability index
J	Jacobian matrix
λ	Asymptotic Lyapunov exponent