Applied Mathematical Sciences

Volume 127

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Inverse Problems for Partial Differential Equations

Third Edition

Springer

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ISSN 0066-5452 ISSN 2196-968X (electronic)
Applied Mathematical Sciences
ISBN 978-3-319-51657-8 ISBN 978-3-319-51658-5 (eBook)
DOI 10.1007/978-3-319-51658-5

Library of Congress Control Number: 2016963146

Mathematics Subject Classification (2010): 35R30, 35R25, 35B60, 35Q61, 35Q86, 35Q91, 35Q93, 31B20, 47A52, 65J22, 65J20, 65M32, 74J25, 78A46, 80A23, 81U40, 86A22, 91G20, 93B07

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To my wife Julie

Most people, if you describe a train of events to them, will tell you what the result would be. They can put those events together in their minds and argue from them that something will come to pass. There are few people, however, who, if you told them a result, would be able to evolve from their own inner consciousness what the steps were which led up to that result. This power is what I mean when I talk of reasoning backward or analytically.

—Arthur Conan Doyle, *A Study in Scarlet*

Preface to the Third Edition

In 10 years after the publication of the second edition of this book, the changing field of inverse problems witnessed further new developments. Parts of the book were used at several universities, and many colleagues and students as well as myself observed several misprints and imprecisions. Some of the research problems from the previous editions have been solved. We comment on major changes.

Chapter 3 is expanded again. The main additions are to Section 3.4: increasing stability for the F. John's Counterexample 3.4.3, Theorem 3.4.4 concerning the increasing (with the growing wave number) stability in the Cauchy problem for the Helmholtz-type equations, and the inclusion of time derivatives into the Carleman estimate of Theorem 3.5.9. In Chapter 4 we added Theorem 4.6.5 on the uniqueness of both a surface coefficient and a boundary coefficient from two sets of the Cauchy data.

Most of the additions are to Chapter 5, starting with some properties of the Cauchy set which replaces the Dirichlet-to-Neumann map in the presence of Dirichlet eigenvalues. New Theorem 5.3.4 gives an increasing stability estimate for the Schrödinger potential in the case of the attenuation. The innovative method of Bukhgeim combining the complex geometrical optics with the stationary phase is exposed in the proof of new Theorem 5.5.2 which replaces old partial uniqueness results for the Schrödinger potential in the two-dimensional case. Theorem 5.7.2 shows the uniqueness of a domain with most general transmission conditions at its boundary of the Schrödinger potential inside this domain providing a canonical form of uniquely identifiable transparent obstacle, which includes the case of a discontinuous conductivity coefficient. Finally, Theorem 5.8.2 provides increasing stability of the conductivity coefficient in the stationary Maxwell system, which cannot be obtained for a scalar conductivity equation.

In Chapter 6 there is new Theorem 6.1.4 showing the increasing (with the wave number) stability of the recovery of the near-field from its far-field pattern. New Section 6.2 contains results on the increasing stability in the inverse source problem for the Helmholtz equation. Theorem 6.3.6

describes recent results on the uniqueness of the Schrödinger potential from the absolute value of the scattering pattern at several wave numbers. In Chapter 8 we obtained new Theorem 8.2.2 guaranteeing a Hölder stability in the inverse source problems with a single set of lateral boundary data for most general cases of systems admitting natural Carleman estimates.

Many exercises have been solved by students, while most of the research problems await solutions. As for the second edition, we found again numerous inaccuracies and misprints which are now corrected. Parts of this new edition (especially concerning increasing stability) were presented by the author when lecturing at the Universidad Autonoma de Madrid, Spain, and at the University of Tokyo, Japan, with its excellent mathematical library which was extremely useful during the work on the new edition. This work was partly supported by Emylou Keith and Betty Dutcher Distinguished Professorship at Wichita State University. The author also thanks the National Science Foundation for the long-term support of his research and of the writing of this revision.

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Preface to the Second Edition

In 8 years after the publication of the first version of this book, the rapidly progressing field of inverse problems witnessed changes and new developments. Parts of the book were used at several universities, and many colleagues and students as well as myself observed several misprints and imprecisions. Some of the research problems from the first edition have been solved. This edition serves the purposes of reflecting these changes and making appropriate corrections. I hope that these additions and corrections resulted in not too many new errors and misprints.

Chapters 1 and 2 contain only 2–3 pages of new material like in Sections 1.5 and 2.5. Chapter 3 is considerably expanded. In particular, we give more convenient definition of pseudoconvexity for second-order equations and included boundary terms in Carleman estimates (Theorem 3.2.1′) and Counterexample 3.2.6. We give a new, shorter proof of Theorem 3.3.1 and new Theorems 3.3.7 and 3.3.12 and Counterexample 3.3.9. We revised Section 3.4, where a new short proof of exact observability inequality is given: proof of Theorem 3.4.1 and Theorems 3.4.3 (Theorem 3.4.2, new edition), 3.4.4, 3.4.8 (Theorem 3.4.9, new edition), and 3.4.9 are new. Section 3.5 is new, and it exposes recent progress on Carleman estimates and uniqueness and stability of the continuation for systems. In Chapter 4 we added to Sections 4.5 and 4.6 some new material on size evaluation of inclusions and on small inclusions. Chapter 5 contains new results on the identification of an elliptic equation from many local boundary measurements (Theorem 5.3.2′, Lemma 5.3.8 (Lemma 5.2.9 of the new edition)), a counterexample to stability, a brief description of recent complete results on uniqueness of conductivity in the plane case, some new results on the identification of many coefficients and of quasilinear equations in Sections 5.5 and 5.6, and changes and most recent results on uniqueness for some important systems, like isotropic elasticity systems. In Chapter 7 we inform about new developments in boundary rigidity problem. Section 7.4 now exposes a complete solution of the uniqueness problem in the attenuated plane tomography over straight lines (Theorem 7.4.1) and an outline of relevant new methods and ideas. In Section 8.2 we give a new general scheme of obtaining uniqueness results based on Carleman estimates and applicable to a wide class of partial differential equations and systems (Theorem 8.2.2) and describe recent progress on uniqueness problem for linear isotropic elasticity system. In Chapter 9 we expanded the exposition in Section 9.1 to reflect increasing importance of the final overdetermination (Theorems 9.1.1 and 9.1.2). In Section 9.2 we expose new stability estimate for the heat equation transform (Theorem 9.2.1, Lemma 9.2.2). New Section 9.3 is dedicated to emerging financial applications: the inverse option pricing problem. We give more detailed proofs in Section 9.5 (Lemma 9.5.5 and proof of Theorem 9.5.2). In Chapter 10 we added a brief description of a new efficient single-layer algorithm for an important inverse problem in acoustics in Section 10.2 and new Section 10.5 on so-called range tests for numerical solutions of overdetermined inverse problems.

Many exercises have been solved by students, while most of the research problems await solutions. Chapter 7 of the final version of the manuscript has been read by Alexander Bukhgeim, who found several misprints and suggested many corrections. The author is grateful to him for his attention and help. He also thanks the National Science Foundation for the long-term support of his research, which stimulated it and the writing of this revision.

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Preface to the First Edition

This book describes the contemporary state of the theory and some numerical aspects of inverse problems in partial differential equations. The topic is of substantial and growing interest for many scientists and engineers and accordingly to graduate students in these areas. Mathematically, these problems are relatively new and quite challenging due to the lack of conventional stability and to nonlinearity and non-convexity. Applications include recovery of inclusions from anomalies of their gravitational fields; reconstruction of the interior of the human body from exterior electrical, ultrasonic, and magnetic measurements; recovery of interior structural parameters of detail of machines and of the underground from similar data (nondestructive evaluation); and locating flying or navigated objects from their acoustic or electromagnetic fields. Currently, there are hundreds of publications containing new and interesting results. A purpose of the book is to collect and present many of them in a readable and informative form. Rigorous proofs are presented whenever they are relatively short and can be demonstrated by quite general mathematical techniques. Also, we prefer to present results that from our point of view contain fresh and promising ideas. In some cases there is no complete mathematical theory, so we give only available results. We do not assume that a reader possesses an enormous mathematical technique. In fact, a moderate knowledge of partial differential equations, of the Fourier transform, and of basic functional analysis will suffice. However, some details of proofs need quite special and sophisticated methods, but we hope that even without completely understanding these details, a reader will find considerable useful and stimulating material. Moreover, we start many chapters with general information about the direct problem, where we collect, in the form of theorems, known (but not simple and not always easy to find) results that are needed in the treatment of inverse problems. We hope that this book (or at least most of it) can be used as a graduate text. Not only do we present recent achievements, but we formulate basic inverse problems, discuss regularization, give a short review of uniqueness in the Cauchy problem, and include several exercises that sometimes substantially complement the book. All of them can be solved by using some modification of the presented methods.

Parts of the book in a preliminary form have been presented as graduate courses at the Johannes Kepler University of Linz, at the University of Kyoto, and at Wichita State University. Many exercises have been solved by students, while most of the research problems await solutions. Parts of the final version of the manuscript have been read by Ilya Bushuyev, Alan Elcrat, Matthias Eller, and Peter Kuchment, who found several misprints and suggested many corrections. The author is grateful to these colleagues for their attention and help. He also thanks the National Science Foundation for the long-term support of his research, which stimulated the writing of this book.

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