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Tomás Caraballo · Xiaoying Han

Applied Nonautonomous and Random Dynamical Systems

Applied Dynamical Systems



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To our families and friends

Preface

During the past two decades, the theory of nonautonomous dynamical systems and random dynamical systems has made substantial progress in studying the long-term dynamics of open systems subject to time-dependent or random forcing. However, most of the existing pertinent literature is fairly technical and almost impenetrable for all except the most dedicated specialists. Moreover, while the concepts and methods of nonautonomous and random dynamical systems are well established, their application to real-world problems is extremely nontrivial. The aim of this work is to provide an accessible and broad introduction to the theory of nonautonomous and random dynamical systems, with an emphasis on applications of the theory to problems arising in the applied sciences and engineering.

The book starts with basic concepts in the theory of autonomous dynamical systems, which are easier to understand and are used as the motivation for the study of nonautonomous and random dynamical systems. Then the framework of nonautonomous dynamical systems is set up, including various approaches to analyze the long-time behavior of nonautonomous problems. Major emphasis is placed on the novel theory of pullback attractors, as it can be regarded as a natural extension of the autonomous theory and allows a larger variety of time-dependent forcing than other alternatives such as skew-product flows or cocycles. Finally, the theory of random dynamical systems and random attractors is introduced and shown to be fairly informative with regard to the study of long-term behavior of stochastic systems with random forcing.

Each set of theory is illustrated by applications to three different models, the chemostat model, the SIR epidemic model, and the Lorenz-84 model, in their autonomous, nonautonomous, and stochastic formulations, respectively. The techniques and methods adopted can be applied to the study of the long-term behavior of a wide range of applications arising in applied sciences and engineering.

Sevilla, Spain
Auburn, USA
October 22, 2016

Tomás Caraballo
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