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Adam Ross Solomon

# Cosmology Beyond Einstein

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the University of Cambridge, UK

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*The effort to understand the universe is one  
of the very few things which lifts  
human life a little above the level of farce  
and gives it some of the grace of tragedy.*

Steven Weinberg, *The First Three Minutes*

*This thesis is dedicated to my parents,  
Scott and Edna Solomon,  
without whose love and support  
I could never have made it this far.*

## Supervisor's Foreword

Einstein's general theory of relativity is one of the most impressive human achievements. It superseded Newton's great work of 1687 to provide us with a new theory of gravitation that extended Newton's theory to domains where velocities could approach that of light and gravitational forces were correspondingly strong. In the limit that speeds are slow and gravity is weak, Einstein's theory is well approximated by Newton's. This is where modern physics departs from the Kuhnian story of scientific 'revolutions'. Old theories are not simply replaced by new ones. Rather, they become limiting cases of the new theory which will hold good in extreme situations where the old one cannot remain consistent. Yet, Einstein's theory went further than merely extending the domain of applicability of our theory of gravity. For the first time it provided a collection of differential equations whose solutions, all of them, describe entire universes. For the first time, cosmology became a science. Physicists could try to solve Einstein's equations in simple cases where there was lots of symmetry to find possible descriptions of our entire astronomical universe. These solutions could then be tested against the astronomical evidence and the subject began to resemble other experimental sciences. Although you cannot experiment on the universe—we only have one universe on display—you can predict correlations that should be observed between different properties of the same mathematical universe and look to see if they exist. In this way, cosmology has become a major scientific enterprise. It makes use of a host of new technologies to create light detectors of previously unimagined sensitivity right across the electromagnetic spectrum and has even begun to see direct evidence of gravitational waves. It has joined forces with elementary particle physicists to share insights and constraints on the behaviour of matter at the highest possible energies. And it has fully exploited the massive increase in computational ability that allows us to simulate the behaviour of large and complicated agglomerations of matter to follow the processes that have led to the formation of galaxies in the universe.

Einstein's theory of general relativity is a spectacular success and agrees with all the observational evidence to extraordinary accuracy. It would be fair to say that the

agreement between theory and observation is so precise that in certain situations, like the binary pulsar's dynamics, it provides us with the surest and most accurate knowledge that human beings have of anything in their experience. So, why do we want to go 'beyond Einstein' in the words of Adam Solomon's thesis title? There are two main reasons. The first is that Einstein's theory has its limits of reliable applicability, just as Newton's theory does. When the density of matter gets too high, as it does near the apparent beginning of an expanding universe and near the centre of black holes, we expect quantum mechanics to modify the character of gravity in a way that will be described by some new theory of quantum gravity. Perhaps this future theory will modify general relativity so as to remove the 'singularities' that presently signal the beginning and of time at the beginning of the universe and the end of time in the inexorable contraction at the centres of black holes? The second reason to look beyond Einstein is of more recent origin. Just 17 years ago, astronomers first discovered that the expansion of the universe smoothly changed gear from deceleration to acceleration about 4 billion years ago. Why this occurred is a big mystery. Three different lines of astronomical evidence find the cause to be a ubiquitous form of energy in the universe—dubbed 'dark energy'—that is gravitationally repulsive. Physicists knew that quantum vacuum energy can have this repulsive effect because of its negative pressure, but no one expected the effects to be dramatically manifested so late in the universe's expansion history. About 70 % of the mass-energy in the universe seems to be in the form of this dark component. What is this dark energy? Is it just a new type of matter field that we have not identified and logged into the energy budget of the universe? This is one line of inquiry that cosmologists explore. The other is to investigate whether there are extensions of Einstein's theory of gravity which introduce new gravitational effects that act to accelerate the expansion of the universe when it is billions of years old and gravity is weak. These new behaviours of gravity need to be well circumscribed. They must not produce new adverse effects locally and in parts of cosmology where observations concur with Einstein's predictions to high accuracy. Adam Solomon's thesis explores a wide class of extensions to Einstein's theory to see whether they can potentially explain the observed acceleration of the universe and account for the existence of galaxies. These extensions cover theories which include a graviton with a non-zero mass and others, like bigravity, where there are two underlying spacetime metrics instead of one. These theories are mathematically more complicated than Einstein's and contain undesirable possibilities that need to be understood and excluded. Adam's thesis contains an elegant and systematic study of these theories, connecting abstract mathematical studies to astronomical predictions and observational tests of the theories. This analysis discovers new ways to solve the equations describing the growth of inhomogeneities and a facility with the observational data and statistical analysis needed to put them to the test. Adam combines a very wide range of mathematical skills and astrophysical understanding to advance our understanding of what a new theory of gravity that



solves the dark energy problem is allowed to look like. The result is a valuable comprehensive study that will lead us a step closer towards the solution of the dark energy problem.

Cambridge, UK  
August 2016

Prof. John D. Barrow

# Abstract

The accelerating expansion of the Universe poses a major challenge to our understanding of fundamental physics. One promising avenue is to modify general relativity and obtain a new description of the gravitational force. Because gravitation dominates the other forces mostly on large scales, cosmological probes provide an ideal testing ground for theories of gravity. In this thesis, we describe two complementary approaches to the problem of testing gravity using cosmology.

In the first part, we discuss the cosmological solutions of massive gravity and its generalisation to a bimetric theory. These theories describe a graviton with a small mass, and can potentially explain the late-time acceleration in a technically natural way. I describe these self-accelerating solutions and investigate the cosmological perturbations in depth, beginning with an investigation of their linear stability, followed by the construction of a method for solving these perturbations in the quasistatic limit. This allows the predictions of stable bimetric models to be compared to observations of structure formation. Next, I discuss prospects for theories in which matter “doubly couples” to both metrics, and examine the cosmological expansion history in both massive gravity and bigravity with a specific double coupling which is ghost-free at low energies.

In the second and final part, we study the consequences of Lorentz violation during inflation. We consider Einstein-aether theory, in which a vector field spontaneously breaks Lorentz symmetry and couples nonminimally to the metric, and allow the vector to couple in a general way to a scalar field. Specialising to inflation, we discuss the slow-roll solutions in background and at the perturbative level. The system exhibits a severe instability which places constraints on such a vector–scalar coupling to be at least five orders of magnitude stronger than suggested by other bounds. As a result, the contribution of Lorentz violation to the inflationary dynamics can only affect the cosmic microwave background by an unobservably small amount.

## Parts of this thesis have been published in the following journal articles:

Following the tendency of modern research in theoretical physics, most of the material discussed in this dissertation is the result of research in a collaboration network. In particular, Chaps. 3–7 were based on work done in collaboration with Yashar Akrami, Luca Amendola, Jonas Enander, Tomi Koivisto, Frank Könnig, Edvard Mörtzell, and Mariele Motta, published in Refs. [1-5] while Chap. 8 is the result of work done in collaboration with John Barrow, published as Ref. [6]. I have made major contributions to the above, in terms of both results and writing.

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<sup>1</sup>Slange var!

# Contents

<b>1</b>	<b>Introduction</b> . . . . .	1
1.1	Conventions . . . . .	5
1.2	General Relativity . . . . .	6
1.3	The Cosmological Standard Model . . . . .	8
1.4	Linear Perturbations Around FLRW . . . . .	12
1.5	Inflation . . . . .	14
	References . . . . .	19
<b>2</b>	<b>Gravity Beyond General Relativity</b> . . . . .	21
2.1	Massive Gravity and Bigravity . . . . .	21
2.1.1	Building the Massive Graviton . . . . .	22
2.1.2	Ghost-Free Massive Gravity . . . . .	28
2.1.3	Cosmological Solutions in Massive Bigravity . . . . .	34
2.2	Einstein-Aether Theory . . . . .	41
2.2.1	Pure Aether Theory . . . . .	42
2.2.2	Coupling to a Scalar Inflation . . . . .	44
2.2.3	Einstein-Aether Cosmology . . . . .	45
	References . . . . .	47
<b>Part I A Massive Graviton</b>		
<b>3</b>	<b>Cosmological Stability of Massive Bigravity</b> . . . . .	55
3.1	Linear Cosmological Perturbations . . . . .	56
3.1.1	Linearised Field Equations . . . . .	57
3.1.2	Counting the Degrees of Freedom . . . . .	60
3.1.3	Gauge Choice and Reducing the Einstein Equations . . . . .	61
3.2	Stability Analysis . . . . .	62
3.3	Summary of Results . . . . .	68
	References . . . . .	69

<b>4</b>	<b>Linear Structure Growth in Massive Bigravity</b> . . . . .	71
4.1	Perturbations in the Subhorizon Limit . . . . .	72
4.2	Structure Growth and Cosmological Observables . . . . .	74
4.2.1	Modified Gravity Parameters . . . . .	74
4.2.2	Numerical Solutions . . . . .	76
4.3	Summary of Results . . . . .	98
	References . . . . .	100
<b>5</b>	<b>The Geometry of Doubly-Coupled Bigravity</b> . . . . .	103
5.1	The Lack of a Physical Metric . . . . .	104
5.2	Light Propagation and the Problem of Observables . . . . .	107
5.3	Point Particles and Non-Riemannian Geometry . . . . .	109
5.4	Summary of Results . . . . .	113
	References . . . . .	114
<b>6</b>	<b>Cosmological Implications of Doubly-Coupled Massive Bigravity</b> . . . . .	117
6.1	Doubly-Coupled Bigravity . . . . .	118
6.2	Cosmological Equations and Their Solutions . . . . .	120
6.2.1	Algebraic Branch of the Bianchi Constraint . . . . .	122
6.2.2	Dynamical Branch of the Bianchi Constraint . . . . .	123
6.3	Comparison to Data: Minimal Models . . . . .	125
6.4	Special Parameter Cases . . . . .	128
6.4.1	Partially-Massless Gravity . . . . .	128
6.4.2	Vacuum Energy and the Question of Self-Acceleration . . . . .	129
6.4.3	Maximally-Symmetric Bigravity . . . . .	131
6.5	Summary of Results . . . . .	131
	References . . . . .	132
<b>7</b>	<b>Cosmological Implications of Doubly-Coupled Massive Gravity</b> . . . . .	135
7.1	Cosmological Backgrounds . . . . .	136
7.2	Do Dynamical Solutions Exist? . . . . .	139
7.3	Einstein Frame Versus Jordan Frame . . . . .	140
7.4	Massive Cosmologies with a Scalar Field . . . . .	141
7.5	Adding a Perfect Fluid . . . . .	143
7.6	Mixed Matter Couplings . . . . .	147
7.7	Summary of Results . . . . .	149
	References . . . . .	151
 <b>Part II Lorentz Violation</b>		
<b>8</b>	<b>Lorentz Violation During Inflation</b> . . . . .	155
8.1	Stability Constraint in Flat Space . . . . .	157
8.2	Cosmological Perturbation Theory . . . . .	162
8.2.1	Perturbation Variables . . . . .	162
8.2.2	Linearised Equations of Motion . . . . .	163

- 8.3 Spin-1 Cosmological Perturbations . . . . . 164
  - 8.3.1 Slow-Roll Limit . . . . . 167
  - 8.3.2 Full Solution for the Vector Modes. . . . . 169
  - 8.3.3 Tachyonic Instability. . . . . 170
  - 8.3.4 What Values Do We Expect for  $\Lambda$ ? . . . . . 173
- 8.4 Spin-0 Cosmological Perturbations: Instability and Observability . . . . . 177
  - 8.4.1 The Spin-0 Equations of Motion. . . . . 178
  - 8.4.2 The Instability Returns . . . . . 179
  - 8.4.3 The Small-Coupling Limit . . . . . 180
  - 8.4.4 The Large-Coupling Limit: The  $\Phi$  Evolution Equation. . . . . 181
  - 8.4.5 The Large-Coupling Limit: CMB Observables . . . . . 184
- 8.5 Case Study: Quadratic Potential . . . . . 186
  - 8.5.1 Slow-Roll Inflation: An Example . . . . . 186
  - 8.5.2 The Instability Explored . . . . . 189
- 8.6 Summary of Results . . . . . 192
- References. . . . . 194
- 9 Discussion and Conclusions . . . . . 197**
  - 9.1 Problems Addressed in This Thesis . . . . . 197
  - 9.2 Summary of Original Results . . . . . 199
    - 9.2.1 Massive Gravity and Bigravity . . . . . 199
    - 9.2.2 Lorentz-Violating Gravity . . . . . 200
  - 9.3 Outlook . . . . . 201
  - References. . . . . 205
- Appendix A: Deriving the Bimetric Perturbation Equations . . . . . 207**
- Appendix B: Explicit Solutions for the Modified Gravity Parameters . . . . . 213**
- Appendix C: Transformation Properties of the Doubly-Coupled Bimetric Action. . . . . 215**
- Appendix D: Einstein-Aether Cosmological Perturbation Equations in Real Space . . . . . 221**
- Curriculum Vitae . . . . . 225**