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Gabriele Sicuro

The Euclidean Matching Problem

Doctoral Thesis accepted by
The University of Pisa, Italy

Springer

Author
Dr. Gabriele Sicuro
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro
Brazil

Supervisor
Prof. Sergio Caracciolo
Dipartimento di Fisica
Università degli Studi di Milano
Milan
Italy

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Stilicidi casus lapidem cavat.

Lucretius
De rerum natura, I.313

To my family and to Elena

Supervisor's Foreword

The Euclidean matching problem is a particular combinatorial optimization problem traditionally considered in the realm of computer science and mathematics.

When, instead of a given instance of an optimization problem, a whole class of problems is considered, according to a suitable probability distribution, methods, ideas, and powerful mathematical tools that physicists have developed in the context of statistical mechanics of systems with frustration and disorder can be applied and have been shown to be very effective also in this area.

For example, the random assignment problem, in which the costs associated to each couple are uncorrelated and identically distributed random variables, has been deeply understood in this way. Its solution represents a sort of mean field approximation for the case, as the one studied here, where correlations are present, for example, because of the geometry of the underlying ambient space.

In this thesis, among other results, a new elegant method is introduced to study the effects of these correlations. It is amusing to discover that a field theory in the continuum is of help to study the asymptotic properties of a discrete number of points in the limit in which this number becomes very large. And the relevant continuum field theory is similar to a reduced version of electromagnetism in which the role of the Gauss law as a constraint is replaced by a transport condition.

Such a similarity allows to study not only the average optimal cost but also the correlation function, under very general conditions, for any distribution of points in Euclidean space (of any dimension). Deep relations among discrete optimization problems, variational methods in the continuum, probability theory, statistical mechanics of disordered systems, and classical field theory are put into evidence in this work. So that readers from different background can find useful inspirations to enlarge their viewpoint.

Milan, Italy
May 2016

Prof. Sergio Caracciolo

Abstract

In the present work, we discuss some connections between combinatorial optimization and statistical physics. In particular, we analyze the so-called *Euclidean bipartite matching problem*, i.e., the matching problem between two different sets of points on an Euclidean domain. We consider the random version of the problem, where the points are independently and identically distributed according to a given probability distribution density. The presence of both randomness and Euclidean constraints makes the study of the average properties of the solution highly non-trivial. We first summarize some known results about both matching problems in general and Euclidean matching problems in particular. We provide a complete and general solution for the one-dimensional problem in the case of convex cost functional. Moreover, we propose an ansatz for the average optimal matching cost in the quadratic case, obtaining both an analytical expression for the finite size corrections in any dimension $d \geq 3$, and the correlation functions in the thermodynamical limit. Finally, we provide, using a functional approach, a general recipe for the computation of the correlation function of the optimal matching in any dimension and on a generic domain.

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