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Andrei-Tudor Patrascu

The Universal Coefficient Theorem and Quantum Field Theory

A Topological Guide for the Duality Seeker

Doctoral Thesis accepted by
University College London, London, UK

Springer

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ISSN 2190-5053 ISSN 2190-5061 (electronic)
Springer Theses
ISBN 978-3-319-46142-7 ISBN 978-3-319-46143-4 (eBook)
DOI 10.1007/978-3-319-46143-4

Library of Congress Control Number: 2016951680

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Illusions commend themselves to us because they save us pain and allow us to enjoy pleasure instead. We must therefore accept it without complaint when they sometimes collide with a bit of reality against which they are dashed to pieces.

Sigmund Freud

To my parents

Supervisor's Foreword

Dualities have been a focus of research in high energy physics over recent decades. They are important because they provide an analytic tool for looking beyond the perturbative domain. Dualities are the tool of choice for many researchers in quantum gravity as well as condensed matter and high energy physics. Non-perturbative effects play an important role in many areas and especially in quantum gravity where quantum fluctuations affecting the topology of spacetime are considered to be common. Therefore, non-perturbative descriptions are required and the ability to derive equations resulting from the various global obstructions that may arise is also needed. This task is related to cohomology theories and various characteristic classes. This book provides an introduction to the field of (co) homology groups and to characteristic classes while focusing on a method which could be used to identify new dualities in physical theories. The discovery of such dualities has relied on astute observation and intuition. The holographic principle and the ER-EPR conjecture are some of the most prominent examples, although, the concept is well known in condensed matter (Kramers–Wannier duality in Ising model), string theory or gauge field theory (Seiberg duality, Montonen–Olive duality, etc.). Despite this, a mathematically well-defined principle which could be used in the search for physical dualities is not known. The advantages of such a concept would be enormous. First, it would allow us to follow a particular set of rules in order to obtain relations between different theories or between different regions of the same theory. Second, it would be of great importance in the study of condensed matter systems where new phases of matter could be detected. Finally, a mathematically well-grounded principle of this type would allow us to extend the catalogue of dualities and introduce new ways of regarding physical theories.

The goal of this work is to study the effects of different coefficient structures in (co)homology on the physical theories described by the respective (co)homologies. This not only gives a new method of analyzing global properties and to express various obstructions in new innovative ways but also to relate formulations of a theory in terms of one coefficient structure with other formulations, expressed by means of other coefficient structures. The main mathematical result employed in this work is the universal coefficient theorem. This theorem allows us to see how changes of the coefficient structure affect the accuracy of various (co)homology groups. It is noteworthy that universal coefficient theorems can be defined for Lie group cohomology as well as for cohomology of various topological spaces. The book explores the effect of the universal coefficient theorem for Lie groups in some situations where such an effect is relevant. The application of choice in this case is the BRST cohomology. Other applications show arguments in favor of the ER-EPR conjecture employing the acyclicity of the circle and results based on twisted cohomology.

Another application discussed in this book is cryptography. Indeed one of the cryptographic protocols in use today is based on elliptic curves. Elliptic cohomology on the other side is based on a cohomology with coefficients in the groups generating elliptic curves. The book discusses also whether the universal coefficient theorem may have some effect on the identification of new attacks on elliptic curve encryption protocols.

Aside from these developments and original work, the book provides the reader with a pedagogical introduction to advanced topological and algebraic concepts. It follows a partially historical path in the development of the required concepts, focusing on how they have been introduced in physics and mathematics. The substantial bibliographic list will most certainly be of use to the researcher in the field.

The book is based on the thesis of Andrei Patrascu who worked as a member of my group at Univirsity College London. However, unusually for a Ph.D. student, this work and the ideas that underlie belong entirely to Andrei with no input from me.

London, UK
April 2016

Prof. Jonathan Tennyson

Abstract

During the end of the 1950s Alexander Grothendieck observed the importance of the coefficient groups in cohomology. Three decades later, he presented his "Esquisse d'un Programme" to the main French funding body. This program also included the use of different coefficient groups in the definition of various (co) homologies. His proposal was rejected. Another three decades later, in the twenty-first century, his research proposal is considered one of the most inspiring and important collection of ideas in pure mathematics. His ideas brought together algebraic topology, geometry, Galois theory, etc. becoming the origin for several new branches of mathematics. Today, Grothendieck is considered one of the most influential mathematicians worldwide. His ideas were important for the proofs of some of the most remarkable mathematical problems like the Weil Conjectures, Mordell Conjectures and the solution of Fermat's last theorem. Grothendieck's dessins d'enfant have been used in mathematical physics in various domains. Seiberg–Witten curves, $N = 1$ and $N = 2$ gauge theories and matrix models are a few examples where his insights are relevant. In this thesis I try to connect the idea of cohomology with coefficients in various algebraic structures to some areas of modern research in physics. The applications are manifold: the universal coefficient theorem presents connections to the topological genus expansion invented by 't Hooft and applied to quantum chromodynamics (QCD) and string theory, but also to strongly coupled electronic systems or condensed matter physics. It also appears to give a more intuitive explanation for topological recursion formulas and the holomorphic anomaly equations. The counting of BPS states may also profit from this new perspective. Indeed, the merging of cohomology classes when a change in coefficient groups is implemented may be related to the wall-crossing formulas and the phenomenon of decay or coupling of BPS states while crossing stability walls. The *Ext* groups appearing in universal coefficient theorems may be regarded as obstructions characterizing the phenomena occurring when BPS stability walls are being crossed. Another important aspect is the existence of dualities. These are the non-perturbative analogue of symmetry transformations. Until now, they were discovered more by accident or by educated guesswork. I show in this thesis that there exists an underlying structure to dualities, a structure that connects them to the number fields used as coefficients in (co)homologies. This observation makes a nontrivial connection between number theory and physics.

Acknowledgments

I wish to thank my supervisor, Prof. J. Tennyson for his advice, understanding and support during the development of my research and the writing of this thesis. I also wish to thank to Prof. J. Iliopoulos for proofreading this work and for advice and support. This work is supported by ERC Advanced Investigator Project 267219.

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