Digital Technologies in Designing Mathematics Education Tasks

MATHEMATICS EDUCATION IN THE DIGITAL ERA
Volume 8

More information about this series at http://www.springer.com/series/10170

Allen Leung · Anna Baccaglini-Frank
Editors

Digital Technologies in Designing Mathematics Education Tasks

Potential and Pitfalls

Springer

Editors
Allen Leung
Department of Education Studies
Hong Kong Baptist University
Kowloon Tong
Hong Kong SAR
China

Anna Baccaglini-Frank
Department of Mathematics
"G. Castelnuovo"
"Sapienza" University of Rome
Rome
Italy

ISSN 2211-8136 ISSN 2211-8144 (electronic)
Mathematics Education in the Digital Era
ISBN 978-3-319-43421-6 ISBN 978-3-319-43423-0 (eBook)
DOI 10.1007/978-3-319-43423-0

Library of Congress Control Number: 2016947472

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG Switzerland

Contents

Introduction

This book is about the role and potential of using digital technology in designing teaching and learning tasks in the mathematics classroom. Digital technology has opened up different new educational spaces for the mathematics classroom in the past few decades and, as technology is constantly evolving, novel ideas and approaches are brewing to enrich these spaces with diverse didactical flavors. A key issue is always how technology can, or cannot, play epistemic and pedagogic roles in the mathematics classroom. The main purpose of this book is to explore mathematics task design when digital technology is part of the teaching and learning environment. What features of the technology used can be capitalized upon to design tasks that transform learners' experiential knowledge, gained from using the technology, into conceptual mathematical knowledge? When do digital environments actually bring an essential (educationally speaking) new dimension to classroom activities? What are some pragmatic and semiotic values of the technology used? These are some of the concerns addressed in the book by expert scholars in this area of research in mathematics education.

Task Design in Mathematics Education and the Growing Interest for Task Design with Digital Technology

More than a decade ago Sierpinska (2003) identified task design as a core research area in mathematics education. She commented that research reports rarely gave sufficient details about tasks for them to be used by someone else in the same way. At the time, few studies justified task choice or identified what features of a task were essential and what features were irrelevant. A growing body of research grounded within different theories, such as the Adaptive Control of Thought learning theory (e.g., Anderson and Schunn 2000), the Theory of Variation applied to teaching and learning (e.g., Runesson 2005), or learning from worked-out examples (e.g., Renkl 2005), suggested that seemingly minor differences in tasks can have significant effects on learning. A growing scientific interest in task design is shown by the hosting, in 2008, of a Topic Study Group (TSG) by the International Congress on Mathematics Education (ICME) entitled *Research and development in task design and analysis* (http://tsg.icme11.org/tsg/show/35) where participants were given the opportunity to experience various tasks, and compare and critique design principles. A number of issues started to emerge. For example, Schoenfeld (2009) advised on the utility of having more communication between designers and researchers, in order to bridge educational research and design. A volume of the *Handbook of Mathematics Teacher Education* was devoted to issues regarding the relationship between teacher education and task design (Tirosh and Wood 2009). A particular issue treated in this context is the role and use of tasks for teacher education purposes, which has recently received particular attention also in a triple special issue of the *Journal of Mathematics Teacher Education* (volume 10, 46) edited by Mason, Watson and Zaslavsky, and in a book edited by Zaslavsky and Sullivan (2011).

The interest in task design in mathematics education has grown more and more, culminating in specific conferences entirely devoted to the topic, including ICMI Study 22. This study "was initiated to produce an up-to-date summary of relevant research about task design in mathematics education and to develop new insights and new areas of relevant knowledge and study." (Watson and Ohtani 2015, p. 3) In the study, task design was described as follows.

> The design and use of tasks for pedagogic purposes is at the core of mathematics education (Artigue and Perrin-Glorian 1991). Tasks generate activity which affords opportunity to encounter mathematical concepts, ideas, strategies, and also to use and develop mathematical thinking and modes of enquiry. Teaching includes the selection, modification, design, sequencing, installation, observation and evaluation of tasks. This work is often undertaken by using a textbook and/or other resources designed by outsiders. (Margolinas 2013, p. 12)

Growing attention has been devoted to tasks that specifically make use of digital technology in the mathematics classroom, since the use of tools and manipulatives in the classroom has been a common pedagogical practice. Drjvers (2012) argued that there are three key factors decisive and crucial to promote or hinder the successful integration of digital technology in mathematics education: the role of the teacher, the educational context, and design. Design is intended not only as the design of the digital technology involved but also as the design of corresponding tasks and activities, and as the design of lessons and teaching in general. Moreover, the scholar argues that emphasis should also be put on the priority of pedagogical and didactical considerations as main guidelines and design heuristics over technology's limitations and properties related to its affordances and constraints.

The design of tasks that make use of digital technology was also discussed in one of the theme chapters in the ICMI Study 22 book (Watson and Ohtani 2015), *Designing Mathematics Tasks: The Role of Tools*, where a tool-based task in the mathematics classroom was described as follows.

> Tools are broadly interpreted as physical or virtual artifacts that have potential to enhance mathematical understanding. A *tool-based task* is seen as a teacher/researcher design aiming to be a thing to do or act on in order for students to activate an interactive tool-based environment where teacher, students, and resources mutually enrich each other in producing mathematical experiences. In this connection, this type of task design rests heavily on a complex relationship between tool mediation, teaching and learning, and mathematical knowledge. (Leung and Bolite-Frant 2015, p. 192)

In that chapter, five tool-based mathematics task design heuristics are proposed to serve as guiding posts to conceptualize mathematics task design in which physical concrete and/or digital tools play central roles in pedagogical activities. This book builds on the existing research on task design in the context of digital technology, exploring some of its potential and pitfalls.

Roles and Potential of Digital Technology in Mathematics Education

This book explores theories and practices of designing mathematics education tasks within what we will call *Dynamic and Interactive Mathematics Learning Environments (DIMLEs)*, coined by Karadag et al. (2011). Rather than showing "how" tasks can be designed using a chosen digital tool, the emphasis of the discussion is on the role and potential of digital pedagogical environments in designing and implementing tasks for the mathematics classroom. Design is seen as the creation of a plan to construct something. It usually starts with some assumptions from which a plan is devised or conjured to achieve some given intended purposes. Different pedagogical or theoretical orientations (the assumptions) and different choices of tools frame pedagogical task design differently. A theme of great interest, which can be studied by contrasting and comparing such framing choices, and which is explored within the chapters of this book, is how different types of task design converge or diverge with respect to the nature and acquisition of mathematical knowledge.

If a *role* is considered a function assumed or a part played by a person or thing in a particular situation, and *potential* considered what is possible in terms of gaining knowledge, the role played by a DIMLE in the mathematics classroom can be multifaceted, and multiple tools can be orchestrated to foster mathematical experiences for the learners. The role of the DIMLE and of tools within it depends on how tasks involving their use are designed and implemented. Meaningful tasks in a DIMLE should capitalize on the affordances, encouraging learners to engage in purposeful and, possibly, non-prescriptive mathematical activities. A challenge in digital task design is to conceive tasks that can extend and amplify pedagogical features present in non-digital environments. For example, an affordance of many DIMLEs is to provide different types of feedback; this can be exploited to establish connections or forms of communication between the learners, the teachers and mathematical knowledge. The potential of a DIMLE can be interpreted with respect to different pedagogical intentions: a *procedural pedagogical intention* that focuses on routine practices for using certain features of the DIMLE, or on establishing algorithmic procedures to generate (desired) results; a *conceptual pedagogical intention* that enables learners to engage in exploring and conceptualizing mathematics. A DIMLE designed with potential to bring about specific the mathematical knowledge may still fail to do so, because whether a DIMLE provides potential or pitfalls for the learners is also very closely related to choices made by the teacher. Indeed, the teacher's pedagogical orientation and mathematical beliefs are key factors in determining how s/he designs and implements tasks developed within the DIMLE. Knowledge/meaning gaps may exist between digital-based mathematical discourses and the intended mathematical content. Potential can become a pitfall or vice versa depending on the ways in which the teacher handles this gap. Inappropriate expectations of the teacher pertaining to a digital tool's potential may turn the digital tool into a pitfall for the learners. Thus, the role and potential of using digital technologies in designing mathematics education tasks are intertwined in a dynamic epistemic complex where mathematical knowledge, pedagogy, and skills are meshed together.

This book, focusing on task design within DMILEs, expands and extends the discussion of the task design theme *Tools and Representations* in ICMI Study 22 (Margolinas 2013; Watson and Ohtani 2015). In the next section, an overview of the structure and the chapters of the book are presented to summarize how the above ideas are threaded together.

Specific Contents of the Book

The chapters in this book explore task design in DIMLEs from multiple perspectives. The authors of each chapter, who are expert scholars in this area of research in mathematics education, employ diverse theoretical dispositions and DIMLEs with different degrees of sophistication in expounding their ideas. The purpose of this book is not to compare and say which DIMLEs are "more conducive" to mathematical learning, whatever that might mean. Whether a (digital) tool is pedagogically significant depends on how the tool is being designed for use in the classroom and how the teachers and students can or cannot actually use it to create meaningful mathematical discourses. The focus of the book is mainly on searching for and ascertaining viable digital tool-based task design approaches for students to engage meaningfully in mathematical experiences. Concerns for the interested readers may lie in the pitfall or potential of tool-based pedagogy and in the "ontological connection" between tools and mathematical knowledge. These issues are addressed, enriching current research literature on task design and on the use of technology in mathematics education, and outlining new research scenarios. The book is divided into four parts that, more specifically, address: (1) theories used to frame and guide digital task design in general; (2) specific kinds of tasks designed for a same type of DIMLE, dynamic geometry environments; (3) design features of five specific

DIMLEs, emphasizing how these features come into play in tasks designed within them; (4) additional issues in digital task design, such as how "variation" principles can be integrated into a digital intervention, how the notions of "feedback" and "discrepancy" can help gain insight into potential and pitfalls of using virtual realizations of physical artifacts, how task design can authentically reflect on the role of digital technologies in solving problems situated in the work place or in daily life.

Part I is mainly about theories. Chapter "Exploring Techno-Pedagogic Task Design in the Mathematics Classroom" explores the idea of techno-pedagogic design in the mathematics classroom. The concept of Mathematics Digital Boundary Object is introduced to stress the role and function of digital technology as a translator and discourse generator between different participants in the mathematics classroom. Task design follows an epistemic nested sequence (Leung 2011) where students' routine algorithmic actions progressively evolve into tool-based situated discourses. The teacher's knowledge plays a critical role in anticipating knowledge/meaning gaps between digital-based mathematical discourses and the intended mathematical content. Chapter "Revisiting Theory for the Design of Tasks: Special Considerations for Digital Environments" discusses Brousseau's notions of 'modes of production' to describe the different types of dialectic interactions between students and the milieu, and of 'ontogenic, didactical and epistemological obstacles' to guide the design of tasks in digital environments (Brousseau 1997). The distinction between 'actual reality' and 'virtual reality' is expounded using Noss, Healy and Hoyles' (1997) terminologies 'pragmatic/empirical' and 'mathematical/systematic'. A digital graphing environment is used as a context to illustrate these theoretical constructs. Chapter "Task Design Potential of Using an Interactive Whiteboard for Implementing Inquiry-Based Learning in Mathematics" makes use of Chevallard's Anthropological Theory of Didactic (Chevallard 1992) which is based on the postulate that a student learns by autonomous adaptation or confrontation in interaction with a milieu. It further frames inquiry-based learning within a milieu through three consecutive processes: mesogenesis, topogenesis, and chronogenesis (Chevallard 2011). As an example, the Interactive White Board is chosen as the milieu for the discussion. Chapter "Designing Technology that Enables Task Design" addresses the question of how technology has been designed to enable task design through interviews with four developers of DIMLEs, carried out with the aim of fostering mathematical learning. Questions ranged from more general ones concerning the purposes and challenges faced in designing the environments to more specific aspects concerning task design, such as the management of processes of instrumental genesis and how feedback is provided. Crucial design aspects of the DIMLEs to be conserved and further developed are disclosed in the interviews. Such aspects include maintaining an appropriate balance between flexibility and constraint as well as addressing issues such as the way in which the environment responds to students' actions.

Part II discusses the design of three different kinds of tasks in a particular type of DIMLE, Dynamic Geometry Environments (DGEs): tasks for geometry assessment, tasks fostering indirect argumentation in Euclidean geometry and tasks in which a real artifact is modeled. Chapter "Designing Assessment Tasks in a Dynamic Geometry Environment" explores designing assessment tasks in DGEs, taking into account how feedback provided by the DGE involves a new dynamic of action/interaction during assessment. This chapter draws on previous work on task design in DGEs (in particular by Laborde 2001) to suggest a framework for identifying and designing different types of assessment tasks according to the specific goals of the teacher. These types of tasks are exemplified using tasks designed for the iPad-based multi-touch Sketchpad Explorer. The DGE assessment tasks presented evaluate different kinds of competencies including digital tools as a main and essential component of the problem-solving process. The authors argue that in order for technology to be a valid part of formal assessment, tasks should assess not only what students know in mathematics, but also what students can do with the technology, like make and test conjectures and, more broadly, students' technological competencies. Chapter "Designing Non-constructability Tasks in a Dynamic Geometry Environment" discusses about the potential offered by specific tasks designed in a DGE leading to production of indirect arguments and proof by contradiction. The main objective is to elaborate on the potentials of designing problems of non-constructability in a DGE with respect to fostering processes of indirect argumentation. The crux of this type of task design is to elicit cognitive conflict through a visual anomaly that to be explained leads students to resort to Euclidean geometry. Using the idea of figural concept (Fischbein 1993) as a "harmony" between a figural and conceptual component, an anomaly can be thought of as a break between the two components (figural and conceptual). It may be possible to restore the harmony within the figural concept by dragging to make a certain configuration vanish or degenerate, or by re-interpreting the figure obtained, rectifying the anomaly. Chapter "The Planimeter as a Real and Virtual Instrument that Mediates an Infinitesimal Approach to Area" focuses on task design involving the digital realization of a model of a specific artifact: the planimeter. The planimeter is a professional tool used for measuring areas of flat regular or irregular shapes. In a teaching experiment, a concrete planimeter and a virtual model built through the DIMLE GeoGebra are used to mediate an infinitesimal approach to area. The planimeter virtualization is displayed to introduce new mathematical knowledge through the construction and the exploration of real situations with the employment of technological tools. The notions of semiotic mediation and didactic cycle by Bartolini Bussi and Mariotti (2008) are used as theoretical framing ingredients in the design of the planimeter tasks.

Part III introduces design features of five specific DIMLEs, providing analyses of examples of tasks designed within each of them. Chapter "Engagement with Interactive Diagrams: The Role Played by Resources and Constraints" establishes theoretical foundations of digital task design for student–textbook–teacher interactions, in particular for analyzing how the designed features of the Interactive Diagrams (IDs) function in actual interaction processes. A semiotic framework is proposed characterized by three types of IDs' functions that address a variety of learning and teaching settings: presentational, orientational, and organizational. The chapter summarizes major design decisions about resources and constraints of interactive texts according to various semiotic functions and the role of the designed constraints and resources of the IDs in processes of engagement with interactive texts. Students' engagements with different types of ID are presented to elaborate the theoretical discussion. Chapter "Everybody Counts: Designing Tasks for *TouchCounts*" introduces the open-ended multi-touch app called Touch Counts (Sinclair and Jackiw 2011) which provides unconventional engagement with the introductory concept of a number and number operations for young children. This chapter describes a series of tasks that have been specifically designed to take advantage of the affordances of TouchCounts. These tasks are analyzed in terms of their novel potential for supporting the development of number, as well as the different functions they draw on in terms of how children are invited to count, operate, and attend to both the ordinal and the cardinal dimensions of number. A guiding feature in the task design is to allow feedback obtained through direct manipulation to serve as evaluation feedback. The authors present and analyze a series of tasks designed and implemented for these two sub-environments in TouchCounts, one for enumerating and the other for operating. The analyses highlight the pragmatic/epistemic values of the tasks, suggesting that the pragmatic value of a task is almost always equivalent to the completion of the task itself, while the epistemic value of each task usually depends strongly on the children's execution of certain bodily actions. Chapter "Designing Innovative Learning Activities to Face Difficulties in Algebra of Dyscalculic Students: Exploiting the Functionalities of *AlNuSet*" presents the DIMLE AlNuSet (Algebra of Numerical Sets), a dynamic algebra environment, designed for students of lower and upper secondary school. Its main function is to afford semiotic multi-representations for algebraic notions such as variable, unknown, algebraic expression, equation, and solution of an equation. The task design focused upon within the chapter deals with how tasks within AlNuSet can be used to promote meaning making in students with low achievement in mathematics, or even diagnosed with Developmental Dyscalculia (DD), including adult learners. The author reports on a case study with a 26-year old DD student. This case study shows that AlNuSet tasks can indeed support the construction of algebraic notions using especially the visual non-verbal and kinaesthetic channels of access to information. Furthermore, the discussion shows how dynamicity is key in the task design, as it fosters students' construction of algebraic meanings and their perception of relations between expressions, also compensating for weak memorization skills when solving algebraic tasks. Chapter "What Can You Infer from This Example? Applications of Online, Rich-Media Tasks for Enhancing Pre-service Teachers' Knowledge of the Roles of Examples in Proving" introduces the online interactive digital platform LessonSketch. The chapter reports and elaborates on the LessonSketch design of a rich-media task "What can you infer from this example?" that addressed pre-service teachers' content and pedagogical knowledge of the status of examples in proving. The focus is on the theoretical and empirical considerations that guided the task design aimed at providing rich learning opportunities for the pre-service teachers to enhance their content and pedagogical knowledge of the interplay between examples and proving, and address some of the challenges involved in the task implementation. The task involves multiple aspects of the environment and use of data collected from actual students represented through scenarios of non-descript cartoon characters. Buchbinder and Zaslavsky's (2009) framework for describing the status of examples in proving is used as the theoretical basis for task design; moreover, the authors present an emergent framework for designing tasks in digital environments for pre-service teachers. Such framework proposes a systematic approach to combining theoretical grounds, empirically tested design features and advanced technological tools through the process of design-research with the aim of creating instructional tasks that address prospective teachers' content and pedagogical knowledge for engaging students in proving.

Part IV collects chapters that discuss a variety of issues in digital task design for mathematics education. Chapter "Supporting Variation in Task Design Through the Use of Technology" describes a digital intervention aimed at fostering algebraic expertise that was built on three principles: crises, feedback, and fading. The principles are retrospectively scrutinized through the Theory of Variation, concluding that the principles share several elements with the patterns of variation: contrast, generalization, separation, and fusion (Marton et al. 2004). The integration of these principles into a digital intervention suggests that technology has affordances with respect to task design with variation. The principles are demonstrated by discussing a sequence of tasks involving quadratic formulas designed in a DIMLE called *Digital Mathematical Environment* (DME, http://www.fi.uu.nl/dwo/en). Chapter "Feedback and Discrepancies of a Physical Toolkit and a Digital Toolkit: Opportunities and Pitfalls for Mediating the Concept of Rotational Symmetry" presents analyses of excerpts of lessons from a lesson study where tool-based tasks were designed to teach the concept of rotational symmetry in Grade 5. The instrumentational approach and the theory of semiotic mediation are used as the theoretical frameworks to analyze and compare the excerpts of a lesson using a tailor-made physical tool and of a lesson using PowerPoint (a digital tool). The discussion focuses on the opportunities and pitfalls that these two tools, given the same tasks assigned, offer and on how the tasks could have and did (or could not or did not) exploit the semiotic potential of the tool used to solve them. In particular, the notions of feedback and discrepancy are theorized and hypothesized in the context of designing and implementing tool-based mathematics tasks. Chapter "Designing for Mathematical Applications and Modelling Tasks in Technology Rich Environments" touches on the important issue of how task design can authentically reflect on the role of digital technologies in solving problems situated in the work place or in daily life. The chapter draws on data sourced from a research and development project that investigated the use of digital technologies in teaching and learning mathematical modeling and applications. Six principles for designing effective modeling tasks are identified; and the instantiation of these principles within classroom practice is illustrated through a classroom vignette. The chapter further reflects on the research needed to further develop understanding of the role of technology in designing mathematical modeling tasks. Chapter "Designing Interactive Dynamic Technology Activities to Support the Development of Conceptual Understanding" discusses how DIMLEs can foster task design for active and transformative learning to support the development of conceptual understanding in mathematics. The author argues how digital task design should follow the experiential learning cycle: concrete experience, reflective observation, abstract conceptualization, and active experimentation in order to create opportunities for students to engage in mathematical discussions. The Building Concepts software is used as a DIMLE illustrating task design rationales for different fundamental mathematical concepts such as fractions, ratios, proportional relations, expressions, and equations. Design guidelines and principles for building concepts activities are proposed and supported with a number of examples. Chapter "Tensions in the Design of Mathematical Technological Environments: Tools and Tasks for the Teaching of Linear Functions" begins by describing a longitudinal study and its theoretical framework that resulted in a rubric to inform the design of tasks that privilege the exploration of mathematical variants and invariants. This rubric is then used as a construct for the post-priori analysis of two tasks introducing the concept of linear functions through the use of different technologies. Conclusions are drawn highlighting subtle tensions that relate to the mathematical knowledge at stake and to the design principles of the underlying technology and task.

Allen Leung
Anna Baccaglini-Frank

References

Anderson, J. R., & Schunn, C. D. (2000). Implications of the ACT-R learning theory: No magic bullets. In R. Glaser (Ed.), *Advances in instructional psychology: Educational design and cognitive science* (Vol. 5, pp. 1–34). Mahwah, NJ: Lawrence Erlbaum Associates.

Artigue, M., & Perrin-Glorian, M.-J. (1991). Didactic engineering, research and development tool: some theoretical problems linked to this duality. *For the Learning of Mathematics, 11*(1), 3–17.

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh & D. Tirosh (Eds.), *Handbook of international research in mathematics education* (pp. 746–783). Lea, USA: Routledge.

Brousseau, G. (1997). *Theory of didactical situations in mathematics: didactique des mathematiques*, 1970–1990. Dordrecht: Kluwer Academic Publishers.

Buchbinder, O., & Zaslavsky, O. (2009). A framework for understanding the status of examples in establishing the validity of mathematical statements. In M. Tzekaki, M. Kaldrimidou & C. Sakonidis (Eds.). *Proceedings of the 33rd conference of the international group for the psychology of mathematics education.* (Vol. 2, pp. 225–232). Thessaloniki, Greece.

Chevallard, Y. (1992). Fundamental concepts in didactics: Perspectives provid-ed by an anthropological approach. *Recherches en Didactique des Mathématiques, special issue*, 131–167.

Chevallard, Y. (2011). La notion d'ingénierie didactique, un concept à réfon-der. Questionnement et éléments de réponse à partir de la TAD. In C. Margolinas, M. Abboud-Blanchard, L. Bueno-Ravel, N. Douek, A. Fluckiger, N. Douek, F. Vandebrouck & F. Wozniak (Eds.), *En amont et en aval des ingénieries didactiques* (pp. 81–108). Grenoble: La pensée sauvage.

Drjvers, P. (2012) *Digital technology in mathematics education: why it works (or doesn't). Plenary talk at the ICME-12*, Seoul, 8 July–15 July 2012, http://www.icme12.org/upload/submission/2017_F.pdf

Fischbein, E. (1993). The theory of figural concepts, *Educational Studies in Mathematics*, *24*, 139–162.

Karadag, Z., Martinovic, D., & Freiman, V. (2011). Dynamic and Interactive Mathematics Learning Environments (DIMLE). In L. R. Wiest & T. Lamberg (Eds.), *The Proceedings of the 33rd annual meeting of the North American chapter of the international group for the psychology of mathematics education*. Reno, Nevada.

Laborde, C. (2001). Integration of Technology in the Design of Geometry Tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning, 6*(3) (pp. 283–317).

Leung, A. (2011). An epistemic model of task design in dynamic geometry environment. *ZDM - The International Journal on Mathematics Education*, *43*, 325–336

Leung, A. & Bolite-Frant, J. (2015). Designing mathematics tasks: The role of tools. In Anne Watson & Minoru Ohtani (Eds.). *Task design in mathematics education: The 22nd ICMI study (New ICMI study series)* (pp. 191–225). New York: Springer.

Margolinas, C. (Ed.) (2013). *Proceedings of ICMI Study 22 Task design in mathematics education*. Oxford, UK. Retrieved July 2013 from http://hal.archives-ouvertes.fr/hal-0083405

Marton, F., Runesson, U., & Tsui, A. B. M. (2004). The space of learning. In F. Marton & A. B. M. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 3–40). New Jersey: Lawrence Erlbaum Associates, INC Publishers.

Noss, R., Healy, L., & Hoyles, C. (1997). The Construction of Mathematical Meanings: Connecting the Visual with the Symbolic. Symbolic'. *Educational studies in mathematics, 33*(2), 203–233.

Renkl, A. (2005). The worked-out-example principle in multimedia learning. In R. Mayer (Ed.), *The Cambridge handbook of multimedia learning*, (pp. 229–246). Cambridge, UK: Cambridge University Press.

Runesson, U. (2005). Beyond discourse and interaction. Variation: a critical aspect for teaching and learning mathematics. *The Cambridge Journal of Education, 35*(1), 69–87.

Schoenfeld, A. H. (2009) Bridging the Cultures of Educational Research and Design. *Educational Designer*, 1(2). Retrieved from http://www.educationaldesigner.org/ed/volume1/issue2/article5

Sierpinska, A. (2003). Research in Mathematics Education: Through a Keyhole. In E. Simmt & B. Davis (Eds.), *Proceedings of the annual meeting of Canadian mathematics education study group*. Acadia University.

Sinclair, N. & Jackiw, N. (2011). *TouchCounts* [software application for the iPad].

Tirosh, D., & Wood, T. (Eds.). (2009). *The international handbook of mathematics teacher education* (Vol. 2). Rotterdam: Sense publishers.

Watson, A. & Ohtani, M. (Eds.) (2015). *Task design in mathematics education: The 22nd ICMI study (New ICMI study series)*. New York: Springer.

Zaslavsky, O., & Sullivan, P. (Eds.). (2011). *Constructing knowledge for teaching secondary mathematics task to enhance prospective and practicing teacher learning*. New-York: Springer.