

Fundamental Theories of Physics

Volume 184

Series editors

Henk van Beijeren
Philippe Blanchard
Paul Busch
Bob Coecke
Dennis Dieks
Detlef Dürr
Roman Frigg
Christopher Fuchs
Giancarlo Ghirardi
Domenico J.W. Giulini
Gregg Jaeger
Claus Kiefer
Nicolaas P. Landsman
Christian Maes
Hermann Nicolai
Vesselin Petkov
Alwyn van der Merwe
Rainer Verch
R.F. Werner
Christian Wuthrich

The international monograph series “Fundamental Theories of Physics” aims to stretch the boundaries of mainstream physics by clarifying and developing the theoretical and conceptual framework of physics and by applying it to a wide range of interdisciplinary scientific fields. Original contributions in well-established fields such as Quantum Physics, Relativity Theory, Cosmology, Quantum Field Theory, Statistical Mechanics and Nonlinear Dynamics are welcome. The series also provides a forum for non-conventional approaches to these fields. Publications should present new and promising ideas, with prospects for their further development, and carefully show how they connect to conventional views of the topic. Although the aim of this series is to go beyond established mainstream physics, a high profile and open-minded Editorial Board will evaluate all contributions carefully to ensure a high scientific standard.

More information about this series at <http://www.springer.com/series/6001>

Michael J.W. Hall · Marcel Reghinatto

Ensembles on Configuration Space

Classical, Quantum, and Beyond

 Springer

Michael J.W. Hall
Centre for Quantum Dynamics
Griffith University
Brisbane, QLD
Australia

Marcel Reginatto
Physikalisch-Technische Bundesanstalt
Braunschweig
Germany

ISSN 0168-1222

Fundamental Theories of Physics

ISBN 978-3-319-34164-4

DOI 10.1007/978-3-319-34166-8

ISSN 2365-6425 (electronic)

ISBN 978-3-319-34166-8 (eBook)

Library of Congress Control Number: 2016940101

© Springer International Publishing Switzerland 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

This Springer imprint is published by Springer Nature

The registered company is Springer International Publishing AG Switzerland

*This book is dedicated to Robyn, and Mila
and Aldo*

Preface

Much effort in theoretical physics goes towards building mathematical models that can describe as wide a variety of physical systems as possible. To build such models, it is necessary to introduce just the right amount of formalism: the mathematics must be able to capture the essential properties of the physical system, while keeping the amount of mathematical structure which does not have a direct physical interpretation to a minimum.

This book is concerned with the description of physical systems in terms of *ensembles on configuration space*. As will be seen, this is an approach which introduces very few physical and mathematical assumptions. As a consequence, the formalism has wide applicability: it can be used to describe physical systems that are deterministic as well as systems subject to uncertainty; discrete systems, particles, and field theories; classical and quantum theories. It also allows for theories that are difficult to formulate using other approaches, such as hybrid quantum-classical theories where there is an interaction between quantum and classical sectors, including the coupling of quantum matter to classical gravity. Finally, it provides insights into classical and quantum physics that not only lead to unified approaches to concepts such as thermodynamics, weak values, locality and superselection, but to novel reconstructions of quantum theory from physical and geometric axioms.

We therefore believe that a detailed account of the formalism and the physics of ensembles on configuration space is valuable in providing a useful (and beautiful) reformulation of existing theories, and in suggesting various generalisations and directions for formulating new theories, and hope that ideas from this book will be incorporated into the standard toolkit of theoretical physicists.

The book is structured into four main parts. Part I deals with general concepts and properties of ensembles on configuration space. Part II examines how quantum mechanics emerges naturally from three very different axiomatic scenarios, based respectively on an exact uncertainty principle, information geometry on configuration space, and local representations of rotations on discrete configuration spaces. Part III develops a theory of hybrid quantum-classical interactions, which overcomes

various no-go theorems in the literature and provides an explicit model of interaction between quantum systems and classical measuring apparatuses. Finally, Part IV extends these ideas to show how quantum fields can be consistently coupled to classical gravity. While much of the material is based on publications by the authors and colleagues over the past 15 years or so, many results are presented here for the first time.

The authors would like to thank a number of people and organisations. We began our collaboration on the physics of ensembles on configuration space by email correspondence, and worked on two papers together before finally meeting face-to-face in Germany in 2001, courtesy of travel funding provided by the Alexander-von-Humboldt Foundation. Our collaboration continued, and was bolstered by again being able to meet in person at the Perimeter Institute in 2009, courtesy of travel funding for a conference organised by Philip Goyal. In addition to these organisations and individuals, MH would also like to thank Howard Wiseman at the Centre for Quantum Dynamics, for permitting time to be spent on this book project, and wife Robyn and children Conan and Seriden for their support. MR is grateful to Hans-Thomas Elze and Dieter Schuch for invitations to the very stimulating DICE and Symmetries in Science meetings, where some of these results were presented for the first time, and would like to thank Ruth and Ava for putting up with the late nights and long hours. Finally, we thank Angela Lahee at Springer for her support throughout all the stages of preparation of this book.

Brisbane, Australia
Braunschweig, Germany
March 2016

Michael J.W. Hall
Marcel Reginatto

Contents

Part I General Properties of Ensembles on Configuration Space

1	Introduction	3
1.1	The Description of Physical Systems in Terms of Ensembles on Configuration Space	3
1.2	Basic Concepts and Examples	4
1.3	Further Examples: Discrete Configuration Spaces	7
1.3.1	Classical Rate Equations	7
1.3.2	Finite-Dimensional Quantum Systems	9
1.4	Fundamental Properties of Ensemble Hamiltonians	10
1.4.1	Conservation of Probability	10
1.4.2	Positivity of Probability	12
1.4.3	Homogeneity	13
1.5	Outline of This Book	14
	References	16
2	Observables, Symmetries and Constraints	19
2.1	Some General Considerations	19
2.1.1	Fundamental Variables and Ontology	19
2.1.2	The Dual Role of Observables	20
2.2	Observables	21
2.3	Examples	24
2.3.1	Position and Momentum Observables	24
2.3.2	Classical Observables	26
2.3.3	Quantum Observables	27
2.4	Eigenensembles, Weak Values and Transition Probabilities	28
2.4.1	Eigenensembles and Eigenvalues	29
2.4.2	Weak Values and Local Densities	30
2.4.3	Transition Probabilities	32
2.5	Symmetries and Transformations	34
2.5.1	Nonrelativistic Particles	35
2.5.2	Rotational Bits	36

2.6	Constraints and Superselection Rules	40
	References	42
3	Interaction, Locality and Measurement.	43
3.1	Introduction.	43
3.2	Joint Ensembles.	44
	3.2.1 Independent Ensembles.	44
	3.2.2 Interaction Versus Noninteraction.	45
	3.2.3 Independence Versus Entanglement	46
3.3	Extending Observables to Joint Ensembles	48
	3.3.1 General Definition	48
	3.3.2 Two Invariance Properties.	50
	3.3.3 Configuration Separability and Strong Separability	51
3.4	Measurement Interactions: A Simple Example	53
3.5	Weak Measurements	55
	3.5.1 Weak Position Measurement	55
	3.5.2 Weak Momentum Measurement.	56
3.6	Measurement-Induced Collapse	57
	References	59
4	Thermodynamics and Mixtures on Configuration Space	61
4.1	Introduction.	61
4.2	Mixtures	62
	4.2.1 General Definition	62
	4.2.2 Classical and Quantum Mixtures	63
	4.2.3 Dynamics and Liouville Equations	65
	4.2.4 Proper and Improper Mixtures	66
4.3	Thermodynamics on Configuration Space	67
	4.3.1 Failure of the Canonical Approach	67
	4.3.2 Thermal Mixtures.	69
	4.3.3 Thermal Weighting Distribution from the Zeroth Law	70
4.4	Quantum Thermal Mixtures.	72
4.5	Classical Thermal Mixtures.	73
4.6	Example: One-Dimensional Classical Systems	75
4.7	Discussion	77
	Appendix 1: Representation of Phase Space Densities by Mixtures	77
	Appendix 2: Solution of the Classical Continuity Equation	80
	References	81
 Part II Axiomatic Approaches to Quantum Mechanics		
5	Quantization of Classical Ensembles via an Exact Uncertainty Principle	85
5.1	Introduction.	85
5.2	An Exact Uncertainty Relation.	86

- 5.3 Derivation of the Schrödinger Equation 88
 - 5.3.1 Classical Mechanics 89
 - 5.3.2 Nonclassical Momentum Fluctuations 90
 - 5.3.3 Exact Uncertainty Principle 91
 - 5.3.4 Independent Subsystems 93
 - 5.3.5 Equations of Motion. 95
 - 5.3.6 Further Remarks on the Exact Uncertainty Relation . . . 96
- 5.4 Derivation of Bosonic Field Equations 97
 - 5.4.1 Classical Ensembles of Fields 98
 - 5.4.2 Momentum Fluctuations and Quantum Ensembles 99
 - 5.4.3 Example: Electromagnetic Field. 102
 - 5.4.4 Example: Gravitational Field 104
- Appendix 1: Hamilton–Jacobi Ensembles 107
- Appendix 2: Proofs of the Theorem and Corollary. 110
- References 112
- 6 The Geometry of Ensembles on Configuration Space. 115**
 - 6.1 Introduction. 115
 - 6.2 Information Metric, Symplectic Structure and Kähler Geometry 116
 - 6.2.1 Information Geometry. 116
 - 6.2.2 Uniqueness of the Information Metric via Markov Mappings 119
 - 6.2.3 Dynamics and Symplectic Geometry 120
 - 6.2.4 Kähler Geometry 122
 - 6.2.5 On the Geometry of Ensembles on Configuration Space 123
 - 6.2.6 Uniqueness of the Kähler Metric via Generalized Markov Mappings 124
 - 6.2.7 Complex Coordinates and Wave Functions 127
 - 6.2.8 Statistical Distance in the Kähler Space 128
 - 6.3 Geometrical Reconstruction of Quantum Mechanics 129
 - 6.3.1 Group of Unitary Transformations 129
 - 6.3.2 Hilbert Space Formulation. 132
 - 6.4 Information Geometry and Quantum Mechanics. 132
- Appendix 1: Symplectic Geometry, Compatibility Conditions, and Kähler Structure 133
- Appendix 2: Generalized Markov Mappings 134
- References 138
- 7 Local Representations of Rotations on Discrete Configuration Spaces 141**
 - 7.1 Introduction. 141
 - 7.2 One Robit. 142
 - 7.3 Two Robits 144

7.3.1 Reduced Phase Space for Two Robits. 145

7.3.2 Two-Robit System with $so(3) \oplus so(3)$ Symmetry. 146

7.3.3 Wave Function Representation and a Condition
for Equivalence to Quantum Mechanics 152

7.4 Discussion 155

Appendix: Invariance of Subsystem Independence

Constraints Under Rotations 156

References 157

Part III Hybrid Quantum-Classical Systems

8 Hybrid Quantum-Classical Ensembles. 161

8.1 Introduction. 161

8.2 Quantum-Classical Ensembles 163

8.2.1 Quantum and Classical Mechanics on Configuration
Space 163

8.2.2 Interacting Quantum and Classical Ensembles 165

8.3 Some General Properties. 167

8.4 Measurement of a Quantum System
by a Classical Apparatus. 174

8.5 Scattering of Classical Particles by Quantum Superpositions . . . 177

8.6 Hybrid Oscillators and Gaussian Ensembles 180

8.6.1 Gaussian Ensembles. 180

8.6.2 Coherent Ensembles. 182

8.7 Hybrid Wigner Functions 183

8.7.1 Definition and Basic Properties 183

8.7.2 Hybrid Wigner Functions for Gaussian Ensembles 186

8.7.3 Covariance Matrix 187

References 190

9 Consistency of Hybrid Quantum-Classical Ensembles 191

9.1 Introduction. 191

9.2 Dynamical Bracket Considerations 193

9.2.1 Two Minimal Conditions 193

9.2.2 Evading No-Go Theorems: An Algebraic Loophole . . . 195

9.3 Generalised Ehrenfest Relations and the Classical Limit 196

9.4 Locality Considerations. 199

9.4.1 Configuration and Momentum Separability 199

9.4.2 Strong Separability. 201

9.4.3 Implications of Strong Separability Violation. 203

9.4.4 Suppression of Strong Separability Violation
via “Classicality” 207

9.5 Measurement Considerations 209

9.5.1 General Measurement Model. 209

9.5.2 Ineffective Decoherence: Improper Mixtures 211

9.5.3 Macroscopic Measuring Devices Revisited 211

9.5.4 Effective Decoherence and Regaining of Strong
Separability 214

9.6 Comparisons with Mean-Field Approach. 216

References 218

**Part IV Classical Gravitational Fields and Their Interaction
with Quantum Fields**

10 Ensembles of Classical Gravitational Fields. 223

10.1 Introduction. 223

10.2 Einstein–Hamilton–Jacobi Equation and Ensembles
for Classical Gravitational Fields 224

10.2.1 Einstein–Hamilton–Jacobi Equation 224

10.2.2 Measure and Probability 226

10.2.3 Classical Ensemble Hamiltonian for the Gravitational
Field. 228

10.2.4 Rate Equation for the Metric Field 229

10.3 Spherically Symmetric Gravity 230

10.4 Ensembles of Black Holes. 232

Appendix 1: The Reconstruction Problem. 235

Appendix 2: Lemaître Coordinates From Gaussian Coordinate
Conditions 237

References 241

11 Coupling of Quantum Fields to Classical Gravity 243

11.1 Introduction. 243

11.2 The Coupling of Classical Gravitational Fields to Quantum
Matter Fields. 247

11.2.1 General Case. 247

11.2.2 Midisuperspace Example: Spherically Symmetric
Gravity 249

11.3 Hybrid Cosmological Model 250

11.3.1 Minisuperspace Hybrid Cosmological Model 250

11.3.2 Midisuperspace Hybrid Cosmological Model 252

11.3.3 Discussion. 254

11.4 CGHS Black Hole in the Presence of a Quantized
Scalar Field. 255

11.4.1 CGHS Black Hole and Classical Massless
Scalar Field. 255

11.4.2 CGHS Black Hole and Quantized Massless
Scalar Field. 257

11.4.3 CHGS Black Hole Formation Through Collapse
of Matter and Hawking Radiation 261

11.4.4 Non-perturbative Approach and the Emergence of Time	266
Appendix: Ground State Gaussian Functional Solution	269
References	271
Appendix A: Variational Derivatives and Integrals.	273
Index	277