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Understanding Complex Systems

Founding Editor: S. Kelso

Future scientific and technological developments in many fields will necessarily depend upon coming to grips with complex systems. Such systems are complex in both their composition – typically many different kinds of components interacting simultaneously and nonlinearly with each other and their environments on multiple levels – and in the rich diversity of behavior of which they are capable.

The Springer Series in Understanding Complex Systems series (UCS) promotes new strategies and paradigms for understanding and realizing applications of complex systems research in a wide variety of fields and endeavors. UCS is explicitly transdisciplinary. It has three main goals: First, to elaborate the concepts, methods and tools of complex systems at all levels of description and in all scientific fields, especially newly emerging areas within the life, social, behavioral, economic, neuro- and cognitive sciences (and derivatives thereof); second, to encourage novel applications of these ideas in various fields of engineering and computation such as robotics, nano-technology, and informatics; third, to provide a single forum within which commonalities and differences in the workings of complex systems may be discerned, hence leading to deeper insight and understanding.

UCS will publish monographs, lecture notes, and selected edited contributions aimed at communicating new findings to a large multidisciplinary audience.

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Modeling Thermodynamic Distance, Curvature and Fluctuations
A Geometric Approach
Preface

This textbook aims to briefly outline the main directions in which the geometrization of thermodynamics has been developed in the last decades. The textbook is accessible to the people trained in thermal sciences but not necessarily with solid formation in mathematics. For this, in the first part of the textbook a summary of the main mathematical concepts is made. In some sense, this makes the textbook self-consistent. The rest of the textbook consists of a collection of results previously obtained in this young branch of thermodynamics. The content is organized as follows.

The first part of the textbook, consisting of four chapters, presents the main mathematical tools. Thus, Chap. 1 presents the historical background of the geometrization of mechanics and thermodynamics. In Chap. 2 some basic concepts are briefly reminded, such as the set theory, the relationships theory, and the theory of simple algebraic structures. Then, the essential concepts used in the theory of linear spaces are introduced. The chapter ends by presenting some results concerning the coordinate transformations and the classification of physical quantities in relation with these transformations. Chapter 3 describes the main types of vectors and the standard method of vector geometrization. Then elementary results of vector calculus are presented. The chapter ends with a very brief introduction to the exterior differential calculus, accompanied by some specific useful results. Chapter 4 describes results of Riemann geometry. Two approaches are presented. The first one is the classic approach. The second approach is based on the theory of differential manifolds and tangent spaces. Both approaches allow defining the tensors of different orders, the Riemann metric and the covariant differentiation, among others. The parallel between the two approaches is very useful for a deeper understanding of concepts.

The second part of the textbook, consisting of five chapters, refers to the application of geometric methods in equilibrium thermodynamics. Chapter 5 summarizes some results of equilibrium thermodynamics. The approach based on potentials is presented, including the standard procedures using the energy representation and the entropy representation. Finally, the extreme principles and the
mathematical conditions for thermodynamic stability are presented. Chapter 6 briefly shows some results of using tools of contact geometry in thermodynamics. Here only the first law of thermodynamics is geometrized. The chapter ends with a few examples of contact currents in thermodynamics. In Chap. 7 an approach based on statistical methods, which allows defining the notions of thermodynamic metric and thermodynamic distance, is presented. The second law of thermodynamics plays a key role in this context. The relationship between the thermodynamic distance and the entropy production is analyzed and links with the Gouy-Stodola theorem are highlighted. Horse-carrot type theorems are also introduced. The manner in which the thermodynamic curvature can be defined is exposed in Chap. 8. The chapter contains examples of calculation of thermodynamic curvature for simple systems. Chapter 9 presents a covariant theory of the thermodynamic fluctuations and analyzes the level of approximation introduced by the classical theory of fluctuations and its Gaussian approximation.

The textbook is a more extensive version of a section of the course of Advanced Thermodynamics presented for master students at the Faculty of Mechanical Engineering, Polytechnic University of Bucharest, starting from the 2003–2004 academic year. The textbook is presented with an ease of access for the readers with education in natural and technical sciences. Thus, most mathematical demonstrations of the theoretical results with higher degree of difficulty are omitted and references for the relevant literature are provided.

As usual, the preparation of such a work is the result of numerous interactions, discussions, consultations, and collaborations. It is a pleasure to remind here some of them. I received special support from colleagues in the European network CARNET (Carnot Network). This cooperation was institutionalized during the years 1994–1999 by two Copernicus projects on thermodynamic topics funded by the European Commission. In particular, I must thank Prof. Bjarne Andresen (University of Copenhagen), Prof. Ryszard Mrugala (University of Torun, Poland), and Dr. Lajos Diósi (Research Institute for Particle and Nuclear Physics, Budapest) whose publications were massively used in the present work. During the elaboration of the material I received technical support from Prof. Peter Salamon (University of San Diego). Also, discussions with Prof. Constantin Udriste (Polytechnic University of Bucharest) allowed a better understanding of the fundamentals of mathematics.

Viorel Badescu
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