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Chiang Kao

# Network Data Envelopment Analysis

Foundations and Extensions

 Springer

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# Preface

How to use fewer resources to generate more outputs and services is a concern of all organizations, including profit-pursuing, government, nonprofit, and all other types of decision-making units (DMUs). This is a problem of efficiency, which has three phases: efficiency measurement, target setting, and goal achievement. Such issues have been studied by economists and management scientists for many years. Since the seminal work of Charnes, Cooper, and Rhodes in 1978, Data Envelopment Analysis (DEA) has become the preeminent nonparametric method for measuring the efficiency of DMUs that apply multiple inputs to produce multiple outputs. In addition to efficiency measurement, the DEA technique is also able to show how much output a DMU can be expected to increase with the current amount of input or how much input can be saved while producing the current level of output by simply increasing its efficiency. In other words, a target for inefficient DMUs to achieve to become efficient is also provided. The DEA technique is thus able to answer the questions that arise in the first two phases of efficiency studies. As a consequence, thousands of papers and dozens of books related to DEA have been published since its introduction in 1978.

A system is usually composed of many subsystems operating interdependently. Conventional DEA only considers the inputs supplied to and the outputs produced from the system in measuring efficiency, ignoring its internal structure. As a result, it is possible that the overall system is efficient, even while all component divisions are not. More significantly, there are cases in which all the component divisions of a DMU have performances that are worse than those of another DMU, and yet the former still has the better system performance. With an eye on solving these problems, many ideas have been extended from the conventional DEA to build models to measure the efficiency of production systems with different network structures, which are referred to as network DEA. However, these ideas are scattered in different publications, which are inconvenient to access, and are difficult for beginners to read, due to a lack of background knowledge. More seriously, some ideas have already been demonstrated to be incorrect. It is thus desirable to have a book that presents the underlying theory, model development,

and applications of network DEA in a systematic way, to give the readers an idea of what should be done when developing a new model. It is also desirable to have a book that discusses the existing models for measuring the efficiency of systems with specific network structures and explores the relationships between the system and division efficiencies. Separating large operations into detailed smaller ones can help identify the divisions that cause inefficiencies in the system. Novel applications are attractive to readers in introducing a method and can also inspire further studies. It will thus be helpful to have a book that describes these applications. Fifteen years after the first appearance of the term network DEA, there is only one edited book of papers from different authors on specific topics related to network DEA. A book that has the abovementioned functions is thus still needed, and so the current work was written to meet this need, with the encouragement of Professor Joe Zhu, Associate Series Editor of Springer's International Series in Operations Research and Management Science.

For systems composed of interrelated divisions, managers need to know how the performances of the various divisions are evaluated and how they are aggregated to form the overall performance of the system. This book provides an advanced exposition on evaluating the performance of systems with network structures. It explores the network nature of most production and operation systems and explains why network analyses are necessary. The discussion of network DEA carried out in this work also clarifies the concept of the conventional whole-unit DEA.

In addition to the conventional connected models in the network DEA, this book highlights a relational model, which is able to show the relationship between the efficiency of a system and those of its component divisions, when the systems being examined have different types of network structures. This relationship shows the extent to which the efficiency of a division impacts that of the system as a whole. The division with the largest effect is the one to which more effort should be devoted, so that the performance of the overall system can be raised in a more effective manner.

This book has several features, as follows. Most models are presented with an associated figure, showing the network structure of the corresponding problem, and examples are also supplied, which make this book appropriate for class use and self-study. An extensive bibliography of current research literature on network DEA is also included, which should be able to inspire researchers to pursue new areas of work. This book is intended for graduate students who are taking courses or writing theses on topics related to performance evaluation, DEA, and multi-criteria decision analysis. It is also suitable for professors and researchers whose research interests are related to the abovementioned topics. Experienced practitioners who want to measure the performance of production, operation, or any other type of DMUs will also find this book helpful for their work.

In writing this book, many of my former doctoral students helped in providing suggestions, comments, and proofreading, including Shih-Nan Hwang of Ming Chuan University (Dean of College of Management), Hong-Tau Lee of National Chin-Yi University of Technology (Vice President), Shiang-Tai Liu of Vanung University (former Dean of College of Management), Jehn-Yih Wong of Ming

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Tainan, Taiwan  
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Professor Kao has also held administrative posts at Cheng Kung University, including Head of Department of Industrial and Information Management, Dean of College of Management, and President of the University. He has published more than fifty papers on DEA in international journals, 20 of which are related to network DEA. Currently, he serves as a member of the Editorial Board for European Journal of Operational Research and an Associate Editor for Omega.