Part I

Scalar-Tensor Theories (Brans–Dicke Theory)

"...Part II is mainly concerned with the introduction of a varying gravitational constant into the framework of general relativity, violating the strong while preserving the weak principle of equivalence (i.e. geodesics for uncharged test particles). To this end a scalar field, ϕ roughly corresponding to κ^{-1} (κ Gravitational constant) is added to the variational principle of general relativity. ..." (Ph.D. Thesis, Abstract).

"...The possibility of a varying gravitational constant has been discussed by Dirac, Jordan, and particularly with respect to Mach's principle by Dicke. The idea is to weaken the strong principle of equivalence through the effective gravitational constant. ... In choosing a variational principle violating the strong principle of equivalence by the introduction of a varying gravitational "constant," it seems desirable to satisfy at least two conditions. First, the variational principle must be similar to the standard Einstein principle, In other words, since the Einstein equations do agree with the observed data fairly well, any extension of the theory might be expected to be formally similar. Second, the variational principle must be consistent with the weak principle of equivalence which is just a generalization of the results of the Eötvös experiment. To satisfy this second condition it will be required that the operational definition of inertial mass be prescribed in a manner formally independent of the structure of the universe. The stress tensor of ponderable matter will be identified formally and interpretatively with that of general relativity. ...

...The variational principle will be thus taken to be

$$\delta_m \int d^4x\sqrt{-g}\left(\phi R + L_m + \omega\frac{\phi_{,\mu}\phi_{,\nu}g^{\mu\nu}}{\phi}\right)$$

Here ϕ has the dimensions of reciprocal gravitational constant, ω is a dimensionless constant number. The field equations associated with this principle become

$$\delta \int d^4x \sqrt{-g}\, L_m = 0$$

$$\phi S_{\alpha\beta} = (T_m)_{\alpha\beta} + \phi_{;\alpha;\beta} - g_{\alpha\beta}\Box\phi - \frac{\omega}{\phi}\left(\phi_{,\alpha}\phi_{,\beta} - \frac{1}{2}g_{\alpha\beta}\phi_{,\lambda}\phi^{,\lambda}\right)$$

$$\omega\left(\frac{2\Box\phi}{\phi} - \frac{\phi_{,\lambda}\phi^{,\lambda}}{\phi^2}\right) = R$$

in which δ_m signifies variation with respect to pertinent matter variable. ... ($S_{\alpha\beta}$ is the Einstein tensor)"

Carl H. Brans: Mach's Principle and a varying Gravitational Constant, Ph.D. Thesis, Princeton University 1961