

# **Solid Mechanics and Its Applications**

Volume 226

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# Contact Force Models for Multibody Dynamics

 Springer

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# Preface

The prediction of the dynamic behavior of multibody mechanical systems typically involves the formulation of the governing equations of motion and the numerical evaluation of their kinematic and dynamic characteristics. This desideratum is reached when all the necessary ingredients that affect the response of the multibody mechanical systems are adequately taken into account. The contact-impact phenomena are among the most important and complex to model because they depend on many factors, such as the geometry of the contacting bodies (surfaces), the material properties, and the constitutive law utilized to represent the interaction among the different bodies that comprise the multibody mechanical systems. Over the past century, the scientific community has demonstrated an increasing interest in solving problems associated with contact-impact phenomena in mechanical systems. However, proper representation of the contact mechanics for modeling and dynamic simulation of multibody systems is still a challenge.

It is well known that the contact-impact phenomena are characterized by abrupt changes in the system state variables, most commonly discontinuities in the system velocities. Other effects directly associated with the impact phenomenon are those of vibrations propagation through the system, local elastic and plastic deformations at the contact zone, frictional energy dissipation, and wear. Furthermore, during an impact event, multibody mechanical systems can exhibit discontinuities in geometry and some material properties, which can be modified or influenced by the impact itself. In order to correctly model and analyze multibody systems encountering contact-impact in general, appropriate contact force models must be selected.

In this book, several compliant contact force models are analyzed within the context of multibody dynamics, in which the main issues associated with the fundamental contact mechanics are also revisited. In particular, various contact force models from linear to nonlinear, purely elastic to dissipative contact force models are presented and their parameters are described. The different numerical methods and algorithms dealing with contact problems in multibody systems are presented and discussed. In dynamic analysis of multibody systems, the deformation is known at every numerical time step from the configuration of the system,

and the forces are evaluated based on the state variables. With the variation of the contact force during the contact period, the dynamic response of the system is obtained by simply including updated forces into the equations of motion. Since the equations of motion are integrated over the period of contact, this approach is quite simple to implement and results in a rather accurate response. This procedure is further improved by including in the time integration scheme, a procedure that controls the time step in order to prevent large penetrations that might numerically develop in the initial contact. This methodology accounts for the changes in the system's configuration during the contact period.

It is known that the process of modeling contact forces plays a key role in the dynamic simulation and analysis of multibody systems that experience contact-impact phenomena. Thus, the contact force model must be computed by using appropriate constitutive laws that take into account material properties of the contacting surfaces, geometric characteristics of the impacting surfaces and, eventually, the rebound impact velocity. In addition, the numerical method used in the determination of the contact force should be stable enough to allow for the integration of the dynamic equations of motion with acceptable efficiency. These characteristics are ensured by using continuous contact force models, in which the forces and penetrations vary in a continuous manner and for which some energy dissipation is included. This approach has the extra benefit of leading to a behavior of the variable time step integrators that could provide more stable response.

In the present work, the gross motion of multibody systems is described by using a two-dimensional formulation based on the absolute coordinates and the contact-impact events are represented by different contact models. In the sequel of this process, the fundamental characteristics of the most popular elastic and dissipative contact force models are described. The similarities of and the differences among the contact force models are investigated for hard and soft contacts by means of the use of high and low values of restitution coefficient for the contacting surfaces. Results for some planar multibody mechanical systems are presented and utilized to discuss the main assumptions and procedures adopted throughout this work. The material provided here indicates that the prediction of the dynamic behavior of mechanical systems involving contact-impact strongly depends on the selection of the contact force model. Overall, this book is aimed to provide a collective source for the multibody dynamics community and beyond on modeling contact forces and dynamics of mechanical systems undergoing contact-impact.

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