

# **Artificial Intelligence: Foundations, Theory, and Algorithms**

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Christian Blum • Günther R. Raidl

# Hybrid Metaheuristics

Powerful Tools for Optimization

 Springer

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*To Gabi and María and our children Júlia,  
Manuela, Marc, and Tobias. Without their  
love and support life would not be the same.*



# Preface

Research in metaheuristics for combinatorial optimization problems has lately experienced a noteworthy shift towards the hybridization of metaheuristics with other techniques for optimization. At the same time, the focus of research has changed from being rather algorithm-oriented to being more problem-oriented. Nowadays the focus is on solving a problem at hand in the best way possible, rather than promoting a certain metaheuristic. This has led to an enormously fruitful cross-fertilization of different areas of optimization, algorithmics, mathematical modeling, operations research, statistics, simulation, and other fields. This cross-fertilization has resulted in a multitude of powerful hybrid algorithms that were obtained by combining components or concepts from different optimization techniques. Hereby, hybridization is not restricted to different variants of metaheuristics but includes, for example, the combination of mathematical programming, dynamic programming, constraint programming or statistical modeling with metaheuristics.

This book tries to cover several prominent hybridization techniques that have proven to be successful on a large variety of applications as well as some newer but highly promising strategies.

A first introductory chapter reviews basic principles of local search, prominent metaheuristics as well as tree search, dynamic programming, mixed integer linear programming, and constraint programming for combinatorial optimization purposes. The following chapters then present in detail five generally applicable hybridization strategies, including exemplary case studies on selected problems. These five approaches are:

- incomplete solution representations and decoders
- problem instance reduction
- large neighborhood search
- parallel non-independent construction of solutions within metaheuristics
- hybridization based on complete solution archives

While these strategies cover many or even most of the hybridization approaches used nowadays, there also exist several others. The last chapter therefore gives a brief overview on some further, prominent concepts and concludes the book.

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<sup>1</sup> <http://rdlab.lsi.upc.edu>



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# Acronyms

ABC	Artificial Bee Colony
ACO	Ant Colony Optimization
AI	Artificial Intelligence
BD	Benders Decomposition
BS	Beam Search
CG	Column Generation
CO	Combinatorial Optimization
CP	Constraint Programming
DP	Dynamic Programming
EA	Evolutionary Algorithm
GA	Genetic Algorithm
GEEN	Global Edge Exchange Neighborhood
GESB	Global Edge Set Based
GLS	Guided Local Search
GMST	Generalized Minimum Spanning Tree
GRASP	Greedy Randomized Adaptive Search Procedures
IG	Iterated Greedy
IKH	Iterated Kruskal Based Heuristic (for the GMST problem)
ILP	Integer Linear Programming
ILS	Iterated Local Search
LAN	Local Area Network
LNS	Large Neighborhood Search
LD	Lagrangian Decomposition
LP	Linear Programming
MMAS	MAX-MIN Ant System
MCSP	Minimum Common String Partition
MDH	Minimum Distance Heuristic (for the GMST problem)
MDS	Minimum Dominating Set
MIP	Mixed Integer Programming
MKP	Multidimensional Knapsack Problem
MWDS	Minimum Weight Dominating Set

NEN	Node Exchange Neighborhood
NSB	Node Set Based
OR	Operations Research
PSO	Particle Swarm Optimization
R2NEN	Restricted Two Node Exchange Neighborhood
RINS	Relaxation Induced Neighborhood Search
SA	Simulated Annealing
SMTWTSP	Single-Machine Total Weighted Tardiness Scheduling Problem
SWO	Squeaky Wheel Optimization
TS	Tabu Search
UDG	Unit Disk Graph
VND	Variable Neighborhood Descent
VNS	Variable Neighborhood Search