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Arne Grenzebach

The Shadow of Black Holes

An Analytic Description

 Springer

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*Phantasie ist wichtiger als Wissen,
denn Wissen ist begrenzt.*

Albert Einstein

Thein
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Preface

All the work presented here covers an analytic geometrical way to construct the shadow of black holes. The shape of the shadow varies in different space-times, i.e., it depends on specific properties of the black hole as, for example, the spin. My aim is to provide calculations as general as possible.

This short book summarizes the scientific results of my doctoral project where I generalized the existing calculations for the shadow of a Kerr black hole. I found analytical formulas for the boundary of the shadow for the general Plebański–Demiański class of stationary, axially symmetric type D solutions of the Einstein–Maxwell equations. As far as I know, such formulas did not exist before not even in the Kerr space-time. With my formulas, it is possible to calculate the shadow for observers at arbitrary positions. In addition, the shadow-plots can be compared with those of a moving observer. If the motion of the observer is in purely radial direction, then the aberration formula of Penrose is recovered from my formulas.

As pointed out in Chap. 1, the existence of the photon region is crucial for determining the shadow of a black hole. This results in the following natural structure of this thesis. In Chap. 2, I discuss in some detail the Plebański–Demiański class of space-times and review relevant properties of its metric. The geometrically important photon region and other interesting regions in the environment of a black hole are considered in Chap. 3. The last chapter, Chap. 4, is dedicated to deduce the formulas that describe the boundary curve of the black hole’s shadow.

Large parts of the scientific results are already published in three papers. The corresponding paragraphs are marked in the following references which refer to my papers Grenzebach et al. (2014, 2015), Grenzebach (2015), respectively. Sentences marked with $[i]$ can be found in total or only slightly modified in the i th paper.

Bremen, Germany
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Arne Grenzebach

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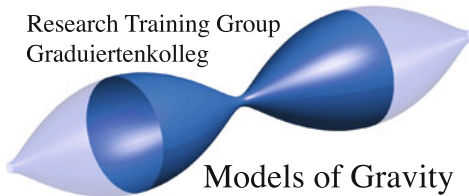


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Models of Gravity

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Abstract

With the upcoming high-resolution observations of the Galactic center, it will be revealed whether our Milky Way hosts a black hole in its center. Due to the strong gravity and the resulting deflection of light, the black hole will cast a shadow and an observed image of the shadow would be a strong evidence for the existence of black holes. It is expected that the *Event Horizon Telescope* or the *BlackHoleCam* project will produce a radio image of the shadow of the central black hole in a few years' time. Therefore, it is about time to advance the theoretical investigations of the shadows of black holes as far as possible, as a basis for evaluating the observational results.

This short book is about an analytic way to describe the shadow of black holes. As an introduction, I present a survey of the attempts to observe the shadow of the black holes in our Galaxy near Sgr A* and in the neighbouring galaxy M87. Black holes are described by metrics of the general Plebański–Demiański class of space-times. All these metrics are axially symmetric and stationary type D solutions to the Einstein–Maxwell equations with a cosmological constant. The space-times are characterized by seven parameters: mass, spin, electric and magnetic charge, gravitomagnetic NUT charge, a so-called acceleration parameter and the cosmological constant.

Based on a detailed discussion of the metrics, I derive analytical formulas for the photon regions (regions that contain spherical lightlike geodesics) and for the boundary curve of the shadow as it is seen by an observer at the given Boyer–Lindquist coordinates in the domain of outer communication. They enable me to analyze the dependency of the shadow of a Kerr black hole on the motion of the observer. For all cases, the photon regions and shadows are visualized for various values of the parameters. The analytical formulas are used to find explicit expressions for the horizontal and vertical angular diameters of the shadow. Finally, these values are estimated for the black holes at the center of our Galaxy and of M87.