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Ching-Shan Chou • Avner Friedman

# Introduction to Mathematical Biology

Modeling, Analysis, and Simulations

 Springer

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