

Models and Methods for Interval-Valued Cooperative Games in Economic Management

Deng-Feng Li

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 Springer

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*To my wife, Wei Fei,
and
to my son, Wei-Long Li*

Preface

Cooperative games with transferable utility are simply called cooperative games in this book. The cooperative game theory is an important branch of the game theory and has been extensively studied. In (crisp or classical) cooperative games, values (or characteristic functions, payoffs) of coalitions of players are expressed with exact values (i.e., real numbers). However, due to uncertainty and information imprecision in real situations, coalitions' values usually have to be estimated. Recently, intervals are used to estimate inherited imprecision or vagueness in coalitions' values, and hereby there appears an important type of cooperative games with interval data (or interval uncertainty), which often are called interval-valued cooperative games for short. A good example may be the bankruptcy problem with interval data. Interval-valued cooperative games are remarkably different from (classical or crisp) cooperative games since their coalitions' values are expressed with intervals rather than real numbers. Recently, some researchers such as S. Z. Alparslan Gök, R. Branzei, O. Branzei, D. Dimitrov, and S. Tijs paid attention to interval-valued cooperative games and have published some articles. However, most of the existing works used Moore's order relation between intervals or interval arithmetic operations, especially Moore's interval subtraction, which is not invertible and hereby usually enlarges uncertainty of the resulted intervals. This case usually is not accordant with real situations. Thereby, inspired by the companion volume *Linear Programming Models and Methods of Matrix Games with Payoffs of Triangular Fuzzy Numbers* (Deng-Feng Li, 2016, Springer, Heidelberg), in this book, we focus on proposing several commonly used and important interval-valued solution concepts of interval-valued cooperative games and hereby developing some simple, practical, and effective models and methods in which the non-invertible interval subtraction or order relation between intervals is effectively avoided.

This book includes three chapters. Chapter 1 proposes the concept of the interval-valued least square solution of interval-valued cooperative games, establishes quadratic programming models and methods for computing interval-valued

least square solutions, and discusses some useful and important properties of interval-valued least square solutions. Chapter 2 studies satisfactory degrees (or ranking indexes) of comparing intervals with the features of inclusion and/or overlap relations and their important properties and proposes the auxiliary satisfactory-degree-based nonlinear programming models for computing interval-valued cores of interval-valued cooperative games and corresponding bisection algorithm. Chapter 3 further expatiates several commonly used and important interval-valued solutions of interval-valued cooperative games and their simplification methods as well as some useful and important properties, including the interval-valued equal division value, the interval-valued equal surplus division value, the interval-valued Shapley value, the interval-valued egalitarian Shapley value, the interval-valued discounted Shapley value, the interval-valued solidarity value, the interval-valued generalized solidarity value, and the interval-valued Banzhaf value. The aim of this book is to develop interval-valued solutions of interval-valued cooperative games and hereby establish their properties, models, methods, and applications, which are remarkably different from the existing studies due to the fact that the non-invertible interval subtraction or order relation between intervals is effectively avoided. I tried my best to ensure that the theoretical models and methods developed in this book are of practicability, simplicity, maneuverability, and universality.

This book is addressed to people in theoretical researches and practical applications from different fields and disciplines such as decision science, game theory, management science, operational research, fuzzy sets or fuzzy mathematics, applied mathematics, industrial engineering, finance, applied economics, expert system, and social economy as well as artificial intelligence. Moreover, it is also addressed to teachers, postgraduates, and doctors in colleges and universities in different disciplines or majors: decision analysis, management, operation research, fuzzy mathematics, fuzzy system analysis, applied mathematics, systems engineering, project management, supply chain management, industrial engineering, applied economics, and hydrology and water resources.

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Last but not least, I would like to acknowledge the encouragement and support of all my friends and colleagues.

Ultimately, I should claim that I am fully responsible for all errors and omissions in this book.

Fuzhou, Fujian, China
October 22, 2015

Deng-Feng Li

Abstract

The focus of this book is to propose several commonly used and important interval-valued solution concepts of interval-valued cooperative games with transferable utility which are called interval-valued cooperative games for short and hereby develop simple and effective models and methods in which the non-invertible interval subtraction or interval order relation is effectively avoided. This book includes three chapters. Chapter 1 proposes the interval-valued least square solutions of interval-valued cooperative games and quadratic programming models and methods as well as properties. Chapter 2 proposes the satisfactory-degree-based nonlinear programming models for computing interval-valued cores of interval-valued cooperative games and corresponding bisection algorithm. Chapter 3 expatiates several interval-valued solutions of interval-valued cooperative games and their simplification methods as well as properties, including the interval-valued equal division and equal surplus division values; the interval-valued Shapley, egalitarian Shapley, and discounted Shapley values; the interval-valued solidarity and generalized solidarity values; and the interval-valued Banzhaf value. The aim of this book is to develop interval-valued solutions of interval-valued cooperative games and hereby establish their properties, models, methods, and applications.

This book is addressed to people in theoretical researches and practical applications from different fields and disciplines such as decision science, game theory, management science, operational research, fuzzy sets or fuzzy mathematics, applied mathematics, industrial engineering, finance, applied economics, expert system, and social economy as well as artificial intelligence. Moreover, it is also addressed to teachers, postgraduates, and doctors in colleges and universities in different disciplines or majors: decision analysis, management, operation research, fuzzy mathematics, fuzzy system analysis, applied mathematics, systems engineering, project management, supply chain management, industrial engineering, applied economics, and hydrology and water resources.

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About the Author



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