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Analysis and Design of Markov Jump Systems with Complex Transition Probabilities

 Springer

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To Liying and Tony

—Lixian Zhang

To my families, teachers, and friends

—Ting Yang

To Fengmei, Lisa, and Michael

—Peng Shi

To my parents and my sister

—Yanzheng Zhu

Preface

The past decades have seen great advances in theories and applications of Markov jump systems (MJSs). The systems can effectively model dynamic systems involving stochastic switching (generally autonomous) subject to a Markov chain. Typical examples include various dynamic systems containing variable parameters, fault-tolerant control systems where abrupt faults occur randomly, networked control systems where network-induced communication imperfections vary in a stochastic way, and so on. Till date, quite a few fundamental control issues, such as stability and stabilization, performance analysis, diverse control methodologies, estimation and filtering, model reduction, fault detection and diagnosis, have been extensively studied and many alluring results have been available in the literature. However, as a crucial factor governing the behaviors of MJSs, the transition probabilities (TPs) are generally considered to be certain, completely known, and time-invariant in most studies. In practice, the assumption of ideal TPs is unrealistic and incomplete TPs are often encountered, especially when adequate samples of the transitions are costly or time-consuming to obtain. Meanwhile, due to the influence of various environmental factors, the time-varying TPs are not rare in real applications.

Considering the important role of TPs in determining the system behaviors and performance, this book collects some of the authors' existing results on MJSs subject to complex TPs as well as the original developments on the stability analysis of MJSs. Three categories of complex TPs will be considered: uncertain TPs where the precise values of TPs are not obtained and only the bounds of TPs are reachable, partially unknown TPs where not all the TPs are available, and time-varying TPs. For time-varying TPs, two descriptive methods will be considered in this book. The first method uses finite piecewise homogeneous TPs, which are governed by a slow switching signal or a stochastic variable subject to a higher-level Markov chain, to present the time-varying TPs. The second method abandons the "memoryless" property of TPs and the resulting TPs display a time-varying feature. The system with memory TPs is also called semi-Markov

jump system (s-MJS). MJSs with ideal TPs, i.e., constant homogeneous TPs, can be deemed as special cases of s-MJSs.

Several basic control issues will be addressed in this book, including stability and stabilization, performance analysis, model reduction, and filter design, etc. According to the type of the complex TPs, the main body of this book is divided into three parts:

- Part I is devoted to the MJSs with partially unknown TPs. Under the stochastic stability notion, the criteria for stability analysis, stabilization, and filtering are presented and then extended to the case where the bound information of some unknown TPs is available.
- Part II focuses on the MJSs with piecewise homogeneous TPs under either nondeterministic switching or stochastic switching. Methodologies that can effectively handle control problems in the scenario are developed, including the one coping with the asynchronous switching phenomenon between the currently activated system mode and the controller/filter to be designed.
- Part III deals with the MJSs with memory TPs. The concept of σ -mean square stability is proposed such that the stability problem can be solved via a finite number of conditions. The systems involved with nonlinear dynamics, which are described via the Takagi–Sugeno (T–S) fuzzy model, are also investigated.

This book is aimed at providing an overview of the recent research advances on MJSs with complex TPs. It can be used in undergraduate and graduate study and is also suitable as a reference for engineers and researchers in this field. Prerequisite to reading this book is elementary knowledge on mathematics, matrix theory, probability, optimization techniques, and control system theory.

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Acronyms and Symbols

ARE	Algebraic Riccati Equation
BRL	Bounded Real Lemma
CCL	Cone Complementary Linearization
MJS	Markov Jump System
MJLS	Markov Jump Linear System
MJNN	Markov Jumping Neural Network
NCS	Networked Control System
NN	Neural Network
TRM	Transition Rates Matrix
TPM	Transition Probabilities Matrix
TR	Transition Rate
TP	Transition Probability
LMI	Linear Matrix Inequality
M'	Transpose of matrix M
\mathbb{R}^n	n dimensional Euclidean space
$\mathbb{R}^{m \times n}$	Set of all $m \times n$ real matrices
\mathbb{N}^+	Set of all positive integers
\mathbb{C}^n	n dimensional complex space
$\mathbb{C}^{m \times n}$	Set of all $m \times n$ complex matrices
$(\Omega, \mathcal{F}, \mathcal{P})$	Ω represents the sample space, \mathcal{F} is the σ -algebra of subsets of the sample space and \mathcal{P} is the probability measure on \mathcal{F}
$E[\cdot]$	Mathematical expectation
*	In symmetric block matrices or long matrix expressions, we use * as an ellipsis for the terms that are introduced by symmetry, for example, $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \triangleq \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$
$diag\{\dots\}$	A block-diagonal matrix $diag\{M_1, \dots, M_n\} \triangleq \begin{bmatrix} M_1 & & 0 \\ & \ddots & \\ 0 & & M_n \end{bmatrix}$

$diag_{(N)}\{M\}$ A block-diagonal matrix $diag_{(N)}\{M\} \triangleq \begin{bmatrix} M & & 0 \\ & \ddots & \\ 0 & & M \end{bmatrix} \in \mathbb{R}^{nN \times nN}$

with $M \in \mathbb{R}^{n \times n}$

$P > 0$ Real symmetric positive (semi-positive) definite matrix P

(≥ 0)

$P < 0$ Real symmetric negative (semi-negative) definite matrix P

(≤ 0)

M_i $M(i)$

$sym(U)$ $U + U'$

I Identity matrix

0 Zero matrix

$\lambda_{\max}(A)$ Maximum eigenvalue of A

$\lambda_{\min}(A)$ Minimum eigenvalue of A

$|\cdot|$ Euclidean vector norm

$L_2[0, \infty)$ Space of square integrable functions on $[0, \infty)$

$l_2[0, \infty)$ Space of square summable infinite sequence on $\{0, 1, 2, \dots\}$

$\|w\|_2$ $\|w\|_2 = \sqrt{\sum_{k=0}^{\infty} |w(k)|^2}$, where $w = \{w(k)\} \in l_2[0, \infty)$

$\|e\|_{E_2}$ $\|e\|_{E_2} = \sqrt{E\left[\sum_{k=0}^{\infty} |e(k)|^2\right]}$, where $e = \{e(k)\} \in l_2((\Omega, \mathcal{F}, \mathcal{P}), [0, \infty))$

$\Pr(A)$ Occurrence probability of the event A

$\inf\{A\}$ Infimum or greatest lower bound of A

$\sup\{A\}$ Supremum or least upper bound of A

$\min\{A\}$ Minimum value of A

class \mathcal{K} Set of continuous and strictly increasing functions that vanish at zero

class \mathcal{K}_{∞} Set of unbounded class \mathcal{K} functions

Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.