

Problem Books in Mathematics

Series Editors:

Peter Winkler
Department of Mathematics
Dartmouth College
Hanover, NH 03755
USA

More information about this series at <http://www.springer.com/series/714>

Leszek Gasiński • Nikolaos S. Papageorgiou

Exercises in Analysis

Part 2: Nonlinear Analysis

Leszek Gasiński
Faculty of Mathematics
and Computer Science
Jagiellonian University
Kraków, Poland

Nikolaos S. Papageorgiou
Department of Mathematics
National Technical University
Athens, Greece

ISSN 0941-3502 ISSN 2197-8506 (electronic)
Problem Books in Mathematics
ISBN 978-3-319-27815-5 ISBN 978-3-319-27817-9 (eBook)
DOI 10.1007/978-3-319-27817-9

Library of Congress Control Number: 2015958668

Mathematics Subject Classification (2010): 00A07; 46-XX; 30LXX, 28-XX, 60G46

Springer Cham Heidelberg New York Dordrecht London

© Springer International Publishing Switzerland 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

Preface

This is the second part of the project initiated with “Exercises in Analysis. Part 1.” In that volume we focused on some classical tools from Analysis. Specifically we considered Metric Spaces, Measure Theory, the interplay between Measure Theory and Topology, and Functional Analysis (Banach Spaces). In the present volume we turn our attention to the main topics of Nonlinear Analysis, which are useful in applications. So, we deal with the following subjects:

1. Function Spaces
2. Nonlinear and Multivalued Maps
3. Smooth and Nonsmooth Calculus
4. Degree Theory and Fixed Point Theory
5. Variational and Topological Methods

Each one of the above topics is a separate chapter. Each chapter starts with a comprehensive presentation of the theory and then has on the average 200 problems together with their solutions. The problems are marked with \star , $\star\star$, and $\star\star\star$ according to the level of difficulty of the problem (\star for easy, $\star\star$ for reasonable, and $\star\star\star$ for difficult). We believe that people using tools of nonlinear analysis will find useful information either in the summary of the theory or in the problems. The topics studied in this volume cover a large part of what is known as Nonlinear Analysis.

Once again the authors express their gratitude to the editor Mrs. Elizabeth Loew for her cooperation and strong moral support in realizing this difficult project and for Krystyna Gasińska and Rafał Kawa for preparing the beautiful drawings of branches decorating the solutions of the problems in both volumes.

Kraków, Poland
Athens, Greece

Leszek Gasiński
Nikolaos S. Papageorgiou

Contents

1	Function Spaces	1
1.1	Introduction	1
1.1.1	L^p -Spaces	1
1.1.2	Lebesgue–Bochner Spaces	8
1.1.3	BV-Functions, Absolutely Continuous Functions, Spaces of Measures	12
1.1.4	Sobolev Spaces	24
1.1.5	Auxiliary Notions	34
1.2	Problems	35
1.3	Solutions	70
	Bibliography	215
2	Nonlinear and Multivalued Maps	217
2.1	Introduction	217
2.1.1	Compact, Completely Continuous, and Proper Maps	217
2.1.2	Multifunctions	224
2.1.3	Maximal Monotone Maps and Generalizations	238
2.1.4	Accretive Maps	246
2.1.5	Miscellaneous Results	250
2.2	Problems	252
2.3	Solutions	284
	Bibliography	407
3	Smooth and Nonsmooth Calculus	409
3.1	Introduction	409
3.1.1	Gâteaux and Fréchet Derivatives	409
3.1.2	Convex Functionals and Variational Inequalities	420
3.1.3	Locally Lipschitz Functions	433
3.1.4	Γ -Convergence and Relaxation	436
3.2	Problems	447
3.3	Solutions	480
	Bibliography	615

4	Degree Theory and Fixed Point Theory	617
4.1	Introduction	617
4.1.1	Degree Theory	617
4.1.2	Metric Fixed Point Theory	635
4.1.3	Topological Fixed Point Theory	639
4.1.4	Order Fixed Point Theory	646
4.2	Problems	654
4.3	Solutions	686
	Bibliography	803
5	Variational and Topological Methods	805
5.1	Introduction	805
5.1.1	Minimization Methods	805
5.1.2	Minimax Methods for Critical Points	815
5.1.3	Morse Theory: Critical Groups	831
5.1.4	Dirichlet Elliptic Problems	836
5.2	Problems	844
5.3	Solutions	885
	Bibliography	1033
	List of Symbols	1035
	Index	1045