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Supersymmetry and Noncommutative Geometry

 Springer

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Preface

The Standard Model of particle physics is an extremely successful theory, describing all currently known elementary particles and their nongravitational interactions. Experimentally, it is tested with previously unencountered precision. However, at the same time it is also clear that at some point it will meet its bounds. For instance, the gravitational force is not included, there are large quadratic corrections to the Higgs mass and it does not account for dark matter. We are therefore in need of a new theory, respecting the various constraints from both experiment and theory, from which the Standard Model emerges as a low-energy limit.

The Standard Model can be beautifully derived from geometrical principles using noncommutative geometry [1, 2]. This includes the Higgs field and the Higgs mechanism. Moreover, the Higgs mass could be predicted in this geometrical framework, but its value turned out to be off (see Sect. 1.2.3 below). At the same time, any prediction of this sort depends on the contents of the chosen noncommutative manifold (e.g. [3]). Application of noncommutative geometry thus gives us new ways to understand the structure of gauge theories in general and the Standard Model in particular. The question is whether it, in addition, can teach us more about reality—via the correct prediction or *retrodiction* of particle masses—than ordinary field theory does. In particular, the hope is that there is a theory that can be considered an extension of the noncommutative Standard Model and that, on top of being phenomenologically viable, yields a sufficiently lower value for the Higgs mass.

The minimally supersymmetric Standard Model (MSSM, Sect. 1.1.1) is a particularly prominent example of physics beyond the Standard Model. Although the question whether supersymmetry is a real symmetry of nature is still open, the merits of the MSSM and models akin alone make them worthwhile to analyze in full detail.

This is the main motivation to search for a theory from noncommutative geometry that describes the MSSM (or something alike), which is the main subject of this book.

To achieve this aim, we will first study the more general question if the spectral action (cf. Eq. 1.21 below) that stems from noncommutative geometry can exhibit

supersymmetry. We do this in Chap. 2. If one is after phenomenologically viable theories of supersymmetry, the question on how to break it again is unavoidable. We therefore turn to this matter in Chap. 3. Finally, we apply the framework developed in Chap. 2 to the almost-commutative geometry that is to give the MSSM in this context in Chap. 4.

Previous attempts to reconcile supersymmetry with noncommutative geometry have been made, see e.g., [4–7], but have not led to conclusive answers. We distinguish ourselves from these approaches in the following ways:

- We try to stay as close as possible to the framework of noncommutative geometry, not digressing into superspace and superfields and the likes.
- All attempts were made prior to the introduction of the spectral action (Eq. 1.21).

Since the latter has proven itself so well in obtaining the Standard Model and since the (predictive) power of the noncommutative method relies heavily on it, we *choose* it to be our action functional and will ask ourselves in Chap. 2 the question “for what noncommutative geometries is the action supersymmetric?” or “what are supersymmetric noncommutative geometries?” This is in contrast to the question “what actions are supersymmetric?” that one typically tries to answer using the superfield formalism. Note the crucial difference here; the intimate connection between an almost-commutative geometry and its associated action forbids us to manually add terms to the latter.

Concerning the prerequisites for reading this book, we assume familiarity with the basic notions in high energy physics (such as action functionals, Lorentz invariance, gauge symmetries) referring to the standard textbooks such as [8–10]. For the two central themes of this book (noncommutative geometry and supersymmetry), references for further reading are included in the main text.

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