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Error Estimates for Well-Balanced Schemes on Simple Balance Laws

One-Dimensional Position-Dependent Models

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Springer

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ISSN 2191-8198 ISSN 2191-8201 (electronic)
SpringerBriefs in Mathematics
ISBN 978-3-319-24784-7 ISBN 978-3-319-24785-4 (eBook)
DOI 10.1007/978-3-319-24785-4

Library of Congress Control Number: 2015953000

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media
(www.springer.com)

To our parents
To Sonia

Foreword

Well-balanced schemes were introduced in the 1990s for solution of hyperbolic conservation laws with source terms. At that time, the idea of taking into account the source term in the numerical fluxes was very new, and several important issues were not understood. The first one was how to build well-balanced schemes. Indeed, the very nonlinear notion of well-balancing, together with the difficulty of consistency, makes it nontrivial to produce such a scheme, even for simple equations. The second issue was what properties are desirable for well-balanced schemes in order to achieve the best compromise between stability and accuracy. During the 2000s, many methods and ideas offered improvements addressing both these issues, thereby generating a collection of practically efficient schemes.

However, third important question remained essentially open until recently: How to devise methods to rigorously analyze these well-balanced schemes? This is, of course, of key importance in order to understand the limitations of known techniques and to improve them further, in particular when resonance occurs. In this monograph, the authors provide a self-contained exposition of useful tools related to this less well understood issue, including their contributions and most recent achievements. Schemes for both scalar laws and semilinear systems, with position-dependent source terms, are analyzed in the spirit of Glimm, with augmented Riemann problems and Lyapunov functionals. Error estimates are established, and a particular form of these estimates concerning the growth in time and the rates in terms of space and time increments offers perhaps the most important characterization of well-balancing that is available at the level of numerical analysis. An exploratory two-dimensional study is also provided, which raises delicate questions. All of the material provided in this book is highly relevant for the understanding of well-balanced schemes and will contribute to future improvements.

Marne-la-Vallee — François Bouchut
May 2015

Preface

The present book originates both from the talks delivered by the first author at several international conferences and from a mini-course given by the second author at BCAM in November 2014. The scope is narrower compared with its companion reference,[1] as most of the aspects related to linear (or weakly nonlinear) kinetic equations have been omitted in order to focus on the rigorous derivation of global error estimates for particular types of (systems of) balance laws in one space dimension.

The monograph presents, in a hopefully attractive and self-contained form, some techniques based on the L^1 stability theory derived at the end of the 1990s by A. Bressan, T.-P. Liu, and T. Yang, which yield original error estimates for so-called well-balanced numerical schemes solving one-dimensional hyperbolic systems of balance laws.[2] Efforts have been focused on a practical assessment of these error bounds, too, either by a wave-front tracking technique or by a simpler Godunov process.

Well-balanced schemes, as they are studied hereafter, mostly rely on a reformulation of the original balance laws as a homogeneous, nonconservative system involving one supplementary steady "fake variable" often denoted $a(x)$. In a strictly hyperbolic regime, a scattering state emerges from the time decay of an extended interaction potential, including the "standing waves" associated with a. Such an asymptotic picture motivates a treatment of source terms, originally suggested by James Glimm,[3] such as "local scattering centers", which we shall apply extensively.

We warmly thank Prof. Enrique Zuazua, who encouraged us to write this manuscript and to submit it for publication in the BCAM Springer Briefs collection.

L'Aquila | Debora Amadori
July 2015 | Laurent Gosse

[1] *Computing Qualitatively Correct Approximations of Balance Laws*, Springer (2013).

[2] *cf.* Marc Laforest, *SIAM J. Math. Anal.* **35** (2004), 1347–1370.

[3] *cf.* J. Glimm and D.H. Sharp, *Found. Phys.* **16** (1986), 125–141.

Contents

Acronyms

BV	Banach space of functions with bounded variation
CFL	Courant–Friedrichs–Lewy
FD	Finite-Differences
FS	Fractional-Step
GNL	Genuinely Non-Linear
LD	Linearly Degenerate
LTE	Local Truncation Error
NC	Non-Conservative
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
RH	Rankine–Hugoniot
TS	Time-Splitting
TV	Total Variation
WFT	Wave-Front Tracking
WB	Well-Balanced