Springer Theses

Recognizing Outstanding Ph.D. Research

Aims and Scope

The series "Springer Theses" brings together a selection of the very best Ph.D. theses from around the world and across the physical sciences. Nominated and endorsed by two recognized specialists, each published volume has been selected for its scientific excellence and the high impact of its contents for the pertinent field of research. For greater accessibility to non-specialists, the published versions include an extended introduction, as well as a foreword by the student's supervisor explaining the special relevance of the work for the field. As a whole, the series will provide a valuable resource both for newcomers to the research fields described, and for other scientists seeking detailed background information on special questions. Finally, it provides an accredited documentation of the valuable contributions made by today's younger generation of scientists.

Theses are accepted into the series by invited nomination only and must fulfill all of the following criteria

- They must be written in good English.
- The topic should fall within the confines of Chemistry, Physics, Earth Sciences, Engineering and related interdisciplinary fields such as Materials, Nanoscience, Chemical Engineering, Complex Systems and Biophysics.
- The work reported in the thesis must represent a significant scientific advance.
- If the thesis includes previously published material, permission to reproduce this must be gained from the respective copyright holder.
- They must have been examined and passed during the 12 months prior to nomination.
- Each thesis should include a foreword by the supervisor outlining the significance of its content.
- The theses should have a clearly defined structure including an introduction accessible to scientists not expert in that particular field.

More information about this series at http://www.springer.com/series/8790

Costantino Budroni

Temporal Quantum Correlations and Hidden Variable Models

Doctoral Thesis accepted by
University of Siegen, Germany

Springer

Author
Dr. Costantino Budroni
University of Siegen
Siegen
Germany

Supervisor
Prof. Otfried Gühne
University of Siegen
Siegen
Germany

ISSN 2190-5053 ISSN 2190-5061 (electronic)
Springer Theses
ISBN 978-3-319-24167-8 ISBN 978-3-319-24169-2 (eBook)
DOI 10.1007/978-3-319-24169-2

Library of Congress Control Number: 2015949295

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

Abstract

This thesis is devoted to the investigation of differences between the predictions of classical and quantum theory. More precisely, we shall analyze such differences starting from their consequences on quantities with a clear empirical meaning, such as probabilities, or relative frequencies, that can be directly observed in experiments.

Different kinds of classical probability theories, or *hidden variable theories*, corresponding to different physical constraints imposed on the measurement scenario are discussed, namely, locality, noncontextuality, and macroscopic realism. Each of these theories predicts bounds on the strength of correlations among different variables, and quantum mechanical predictions violate such bounds, thus revealing a stark contrast with our classical intuition.

Our work starts with the investigation of the set of classical probabilities by means of the correlation polytope approach, which provides a minimal and optimal set of bounds for classical correlations. In order to overcome some of the computational difficulties associated with it, we develop an alternative method that avoids the direct computation of the polytope and we apply it to Bell and noncontextuality scenarios showing its advantages both for analytical and numerical computations.

A different notion of optimality is then discussed for noncontextuality scenarios that provide a state-independent violation: Optimal expression are those maximizing the ratio between the quantum and the classical value. We show that this problem can be formulated as a linear program and solved with standard numerical techniques. Moreover, optimal inequalities for the cases analyzed are also proven to be part of the minimal set described above.

Subsequently, we provide a general method to analyze quantum correlations in the sequential measurement scenario, which allows us to compute the maximal correlations. Such a method has a direct application for computation of maximal quantum violations of Leggett–Garg inequalities, i.e., the bounds for correlation in a macroscopic realist theories, and it is relevant in the analysis of noncontextuality tests, where sequential measurements are usually employed.

Finally, we discuss a possible application of the above results for the construction of dimension witnesses, i.e., as a certification of the minimal dimension of the Hilbert spaces needed to explain the arising of certain quantum correlations.

Supervisor's Foreword

The founding fathers of quantum mechanics, Einstein, Bohr, Schrödinger, Heisenberg, to mention only a few, who struggled to come up with the "definitive" theory we use nowadays, were well aware of the strangeness of the theory and the necessity for an *interpretation* of it. The result was the so-called *Copenhagen interpretation*: a set of rules to extract quantitative predictions for well-defined experimental situations and avoid some of the idealization implicit in the classical theory. Such an attitude towards quantum theory has been summarized by Mermin in the (in)famous "*Shut up and calculate!*". According to him, the bad reputation of foundational problems started with the subsequent generation of physicists that were "*firmly—at times even ferociously—committed to the position that there is really nothing peculiar about the quantum world at all*".

The most important open problem is arguably the *hidden variable problem*, formulated by Einstein, Podolsky and Rosen (EPR) in their 1935 paper. This concerns the possibility of completing quantum mechanics with additional hidden variables, in order to reinterpret quantum probabilities as averages on a phase space, in a manner reminiscent of classical statistical mechanics. EPRs original argument was developed further by Bell who showed that any completion of quantum mechanics with a hidden variable theory satisfying the locality assumption, i.e., the existence of a finite speed at which causal influences can travel, must also obey some bounds on the possible strength of correlations. These bounds are nowadays known as Bell inequalities and they are violated in quantum mechanics, showing a contradiction between testable predictions of quantum mechanics and local hidden variable theories. A similar work of the same years is the one by Kochen and Specker, who showed a contradiction between noncontextual hidden variable theories, where the value of a certain physical quantity is independent of the measurement context, and quantum mechanics. A third fundamental result is the one by

Tsirelson, who introduced a Bell-like inequality for quantum correlations, showing that also quantum mechanics obeys fundamental limitations with respect to more general post-quantum theories.

Such results have been largely ignored by the physics community for several years, until the 1980s when the information theoretic revolution was brought into quantum theory. Then, the "spooky" quantum effects of Einstein, Schrödinger, and Bell became useful resources for new approaches to information processing tasks. The rise of quantum-information ideas was accompanied by an enormous progress in the experimental control and manipulation of physical system at the quantum level, such as single atoms and photons, opening realistic possibilities for applications such as quantum computation, quantum simulation and secure communication, as well as for quantum-enhanced measurements.

Costantino Budroni joined our group in spring 2012, and focused his PhD on these fundamental problems of quantum mechanics and quantum information. This written thesis presents a coherent set of original and outstanding results published as ten articles in various journals. All of them are devoted to the investigation of differences between quantum theory, classical probability theories, and possible post-quantum theories, together with their applications for information processing tasks.

The first part of the work focuses on the stark contrast between the correlations in the outcomes of measurements predicted by quantum theory and the corresponding sets of correlations allowed in theories satisfying different sets of physical constraints, namely, locality (as considered by Bell), noncontextuality (as considered by Kochen and Specker) and macroscopic realism (as considered by Leggett and Garg). Here the correlation polytope approach is adopted to characterize minimal and optimal sets of bounds for classical probabilities. In order to overcome the associated computational difficulties, the thesis presents new approaches, proven to be advantageous in both analytical and numerical computations.

In the second part, a characterization of quantum probabilities is given for the temporal scenario, namely, for sequences of projective measurements performed on the same system. As mentioned above, an important problem is to understand how and why correlations are limited. The problem has been extensively investigated in multipartite Bell scenarios, with the discovery of fundamental limitations like the Tsirelson bound and has given rise to a lively debate on the origin of it. The thesis work on the existence of fundamental bounds for temporal correlations, closely related to multipartite scenarios, but with fundamental differences (e.g., in some cases, they depend on the dimension), opens a new line of research for identifying the fundamental principles of quantum correlations in general scenarios.

Finally, some applications of the above results, such as quantum dimension witnesses, are discussed. The thesis ends with a brief chapter in which all the main results are coherently connected and summarized.

In my opinion, this is an excellent thesis which contains some important original results that will have long standing influence in the fields of foundations of quantum theory and quantum information. This thesis has been awarded the prize for international young researchers from the University of Siegen in 2014, and I am happy that it appears now in the renowned book series Springer Theses.

Siegen
September 2015

Prof. Otfried Gühne

Contents