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Alain Haraux · Mohamed Ali Jendoubi

The Convergence Problem for Dissipative Autonomous Systems

Classical Methods and Recent Advances



Alain Haraux
Laboratoire Jacques-Louis Lions
Sorbonne Universités, UPMC Univ Paris 06,
CNRS, UMR 7598
Paris
France

Mohamed Ali Jendoubi
Institut Préparatoire aux Etudes Scientifiques
et Techniques
Université de Carthage
La Marsa
Tunisia

and

Faculté des sciences de Tunis, Laboratoire
EDP-LR03ES04
Université de Tunis El Manar
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Preface

The initial motivation for the present text was the desire to provide an up-to-date translation of a monograph written in French by the first author, taking account of the more recent developments in infinite dimensional dynamics based on the Łojasiewicz gradient inequality.

While preparing the project, it appeared that it would not be easy to cover the entire scope of the French version within a reasonable amount of time owing to the fact that the non-autonomous systems require sophisticated tools which have undergone major improvements during the past decade.

In order to limit the present work to a modest size and to make it available to readers without unnecessary delay, we decided to produce a first volume dedicated to the so-called convergence problem for autonomous systems of dissipative type. We hope that this volume will help the interested reader to make connection between the relatively simple background developed in the French monograph and the technical specialized literature on the convergence problem, which has expanded rather rapidly in recent years.

Paris, France
La Marsa, Tunisia

Alain Haraux
Mohamed Ali Jendoubi

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