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Self-Oscillations in Dynamic Systems

A New Methodology via Two-Relay Controllers

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ISSN 2324-9749 ISSN 2324-9757 (electronic)
Systems & Control: Foundations & Applications
ISBN 978-3-319-23302-4 ISBN 978-3-319-23303-1 (eBook)
DOI 10.1007/978-3-319-23303-1

Library of Congress Control Number: 2015953028

Mathematics Subject Classification (2010): 93B12, 93C10, 93C80, 34D20, 34C25

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

We dedicate this book with love and gratitude to
Luis's wife Erica, his son Gabriel, and his daughter Elisa,
Igor's wife Natasha,
Leonid's wife Millie,
Rafael's family: his son Rafa, his daughter Erika, and his wife Judith

Preface

This book began with a question asked by Professor Luis T. Aguilar to Professor Leonid Fridman in 2005: How can I generate oscillations of low frequency and particular amplitude using variable structure control?

Coincidentally, this question was asked in the right place and at the right time, because in 2005, Professors Igor Boiko and Leonid Fridman completed their research on the second-order sliding mode control algorithms in frequency domain [13, 14, 16–18], resulting in the possibility of calculating the amplitude and frequency of chattering in systems with second-order sliding mode controllers. They discovered that describing functions (DF) of the second-order sliding mode control algorithms could shift the point characterizing the oscillatory mode resulting from chattering to the second and third quadrants of the complex plane. With the discovery of this property, a straightforward logical conclusion could be made that the problem of generation of self-oscillations (SO) with desired amplitude and frequency could be defined as an inverse problem with respect to the one previously studied. Motivated by this question, Professor Rafael Iriarte found his subject of research in this area too.

Usually, the DF of a single-valued nonlinearity is located on the negative part of the real axis of the complex plane. So for the design of SO in such a situation, only dynamic compensators can be employed, but the possibility of compensators to shape the Nyquist plot of the plant is very limited.

Therefore, the idea that *the controller itself could be designed in such a way that its DF (negative reciprocal of the DF) might be placed in any desired point of the complex plane* was conceived. This idea serves as the basis for the main subject of this book.

In this book, the two-relay controller (TRC) is proposed, which is intended for the generation of SO in dynamic systems. A remarkable feature of this controller is the possibility, with a simple change of controller gains, for one to produce the DF in every phase angle between 0 and 360°, which corresponds to the crossing of the Nyquist plot of the plant and the negative reciprocal of the DF of the controller in any desired point. This point would define the SO produced in the system containing the plant and the controller. The design procedures for TRC are proposed using three different methodologies based on the following: DF, Poincaré maps, and locus of a perturbed relay system (LPRS) method. Three strategies of robustification of generated SO are also proposed. The theoretical results are illustrated by experiments on SO generation in four underactuated systems: wheel pendulum, Furuta pendulum, three-link robot, and three-degrees-of-freedom (3-DOF) helicopter. The experiments are recorded with available video recordings presented in the following web links:

– https://www.youtube.com/watch?v=t_1DcUdwFGE
– https://www.youtube.com/watch?v=MwXVQXlbJMQ

Acknowledgments

We wish to thank our friends and colleagues Professors Leonid Freidovich, Luis Martínez-Salamero, and Iván Castellanos and Drs. Alejandra Ferreira and Antonio Estrada.

The work was partially supported by CONACYT under grant 132125, SIP-IPN Grants, PAPIIT under Grant IN113613 and IN112915, DGAPA PASPA Program, and RIFP Project No. 12310 of the Petroleum Institute.

Tijuana, Mexico — Luis T. Aguilar
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June 2015 — Rafael Iriarte

Notations and Acronyms

TRC	Two-relay control
SO	Self-oscillation
DF	Describing function
SOSM	Second-order sliding modes
HOSM	High-order sliding modes
LPRS	Locus of a perturbed relay system
DOF	Degree(s) of freedom
AOS	Asymptotic orbital stability

$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	The set of all n-dimensional vector with real numbers
$\mathbb{R}^{m \times n}$	The set of all $m \times n$ matrices with real elements
$\mathbb{C}$	The set of complex numbers
j	Imaginary unit
$d(p, S)$	Distance between the point p and the set S ($\inf_{x \in S} \|p - x\|$)
$q \in \mathbb{R}^n$	Joint position vector
$\dot{q} \in \mathbb{R}^n$	The time derivative of the joint position vector
c_1, c_2	Two-relay controller gains
$\boldsymbol{A}_1$	Amplitude of the oscillation
ω	Frequency of the oscillation
$\boldsymbol{\Omega}$	Particular or desired value of frequency of the oscillation
t	Time
$N(A, \omega)$	Describing function depending on the amplitude and frequency of the oscillation
s	Frequency domain complex variable $s = j\omega$
$W(s)$	Transfer function
ξ	Ratio of the two-relay controller gains
T	Period of a signal
η	Actuated states vector
ν	Unactuated states vector
$\eta_1^\star,\ \nu^\star$	Fixed point of Poincaré map

t_2, $\bar{t}_2$	Hypothetical boundary crossing times in Poincaré map construction
σ_0	Constant term in the error signal
σ_p	Sum of periodic terms of Fourier series of the error signal
u_0	Constant term in the control signal
u_p	Sum of periodic terms of Fourier series of the control signal
y_0	Constant term in the output signal
y_p	Sum of periodic terms of Fourier series of the output signal
θ	Asymmetric duty in two-relay controller
$L(\omega, \theta)$	Operative function for LPRS computation
$J(\omega)$	LPRS complex function
$A_L(\omega)$	Modulus of the transfer function $\lvert W(j\omega)\rvert$
A_u	Amplitude of the control signal (first harmonic)
γ	Constant value that provides a fraction of the period T of a signal $t = \gamma T$, $\gamma \in [-0.5; 0.5]$
a_u	Amplitude of the control signal

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