The Scottish Café, birthplace of the Scottish Book problems, as it appeared in a postcard from the early 1970s. The Scottish Café is on the right, with the Café Roma on the left. According to Stanisław Ulam, this scene has changed little from the period preceding World War II

R. Daniel Mauldin

The Scottish Book

Mathematics from the Scottish Café

with Selected Problems from The New Scottish Book

Second Edition

Birkhäuser

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Preface to the Second Edition

The Scottish Book, wrapped in the mists of the past, is a legend in the mathematics world. Its fascinating story and the legendary figures who formulated the problems in the book continue to hold our attention. It represents the best of café mathematics, an informal, free-wheeling style of mathematical conversation and interaction that seems almost lost today. For the solution to some of the problems, prizes were offered, ranging from the famous live goose, a bottle of whiskey of measure > 0, one kilo of bacon to one small beer.

One can imagine the atmosphere at the Scottish Café while the problems were being formulated. We should take to heart the playfulness and enjoyment that is on display, a good model for encouraging and stimulating mathematics at any age. One can also see that a few of the problems must have been stated after spending some time drinking tea or perhaps a brandy or two. In his book *Adventures of a Mathematician*, Ulam describes what a session (one lasting at least seventeen hours) at the café might be like and sketches some of the central figures. During the years I knew and worked with Ulam, he loved the café mode of discussion. Gian-Carlo Rota gives a detailed description of this in his article 'The Lost Café', in *From Cardinals to Chaos*, edited by N. G. Copper (Cambridge University Press, 1989). The entire school of mathematics in Lwów is wonderfully presented by Roman Duda in *Pearls From a Lost City* (American Mathematical Society, 2014), a translation of his 2008 Polish version. The tragedy that befell so many of the figures around the book has been traced many times, including the article by Joanna Diane Caytas, 'Survival of the Scottish Book: A Phoenix from the Holocaust of Polish Mathematics', available on the internet. There is even a collection of poems about the book by Susana H. Case, *The Scottish Café* (Slapering Hot press, 2002).

The problems and the ideas behind continue to effect mathematics today. In the 35 years since the first edition of the book, many more problems have been solved or partially solved. But even today, quite a few remain unsolved. In view of this, I decided to gather new commentaries and update some of the old ones. The appendices of this edition include a list of the unsolved and partially solved problems together with those that have no commentary, a list of unsolved prize problems, a list of problems posed by each author, and a list of problems by subject

area. Besides correcting many errors in the first edition, for clarification a few changes in Ulam's translation of the original Scottish Book have been made.

In addition to some of the lectures given at the 1979 Scottish Book conference, this edition also includes a brief history of Wrocław's *New Scottish Book* and some selected problems from it.

This edition would not be possible without the generous contributions and suggestions of the commentators. I offer them my heartfelt gratitude. I want to thank Kirby Baker, Larry Lindsay, Bill Bernard, and Sue DeMeritt for their assistance in getting this project underway. I thank Al Hales, Joe Buhler, and Jan Mycielski for their counsel. I thank Allen Mann, Christopher Tominich, and Benjamin Levitt at Birkhäuser for their support during the preparation of this new edition. Finally, I thank Diana, my wife, for supporting me throughout this long project.

It is my sincere hope that this collection will bring to the reader, as it has to me, many hours of enjoyment and an image of what must have been a most wonderful place, The Scottish Café.

San Diego, CA, USA R. Daniel Mauldin

Preface to the First Edition

Once while working on a problem, someone was kind enough to point out that my problem was in the "Scottish Book." I had not heard of the Scottish Book and certainly did not realize that this book had no connection with Scotland. But, since that introduction, I've become more and more aware of the magic of the mathematicians and the mathematics involved in its birth.

The Scottish Book offers a unique opportunity to communicate with the men (no women were on the scene, as I understand it) and ideas of a time and place, Lwów, Poland, which have had an enormous influence on the development of mathematics. The history of the Scottish Book as detailed in the following lectures by Ulam, Kac, and Zygmund provides amazing insights into the mathematical environment of Lwów before World War II.

There are many collections of problems, but this set has become world-renowned. Perhaps, a primary reason for this renown is that the problems are clearly and simply formulated, accessible to the general mathematical community, and yet strike at the heart of the concepts involved.

It is my pleasure and honor to edit this version of the Scottish Book, which includes a collection of some of the talks given at the "'Scottish Book Conference"' held at North Texas State University in May of 1979. The purpose of the conference was to examine the history, development, and influence of the Scottish Book. As John Oxtoby toasted at the conference, there was a "'condensation of Poles"' at this conference. Among them were some of the original contributors to the Scottish Book, Professors Ulam, Kac, and Zygmund. Their edited talks appear here, together with the talk given by one close to them in spirit and collaboration, Professor Paul Erdós. Also presented here is a talk by a member of a younger generation, Professor Andrzej Granas, in which one problem of Schauder is discussed with its many-faceted implications and connections. It should come as no surprise that the conference was held in Texas; the mathematical similarities between the Texas school and the Polish school have long been noted, beginning with the fact that the first American to publish in *Fundamenta Mathematicae* is R. L. Moore.

From a glance at the problems, one sees that they cover a wide range of mathematics. I think this simply reflects the wide interests of the unusual group,

which assembled the collection. The problems are concentrated in the areas of summability theory, functional and real analysis, group theory, point set topology, measure theory, set theory, and probability. It is likewise easy to confirm that some of the contributors to the Book were, as R. H. Bing toasted, the "'leading lights"' in these fields.

I have attempted to obtain an appropriate commentary for each of the problems, although quite a few of the problems remain without comment. Some of these, as well as a number of problems with comments, remain unsolved to this day. For others I simply failed to get an appropriate expert comment (I would be grateful for contributions from readers of this edition).

Following a problem there may appear the word *Addendum*. This indicates a comment that was entered into the Scottish Book during the time when the problems were being collected in Lwów. Later commentaries, remarks, and solution to problems which are presented here for the first time follow the original addenda.

The problems and original addenda appear here essentially as they have in the two earlier English-language editions of the Scottish Book, both edited (and one produced) by Stanisław Ulam. The first, in 1957, was a mimeographed version of Ulam's own translation from the original languages in which the problems were inscribed in the Book (mostly Polish), which he distributed on personal request from his professional base at Los Alamos National Laboratory. By 1977, the volume of requests addressed to both Professor Ulam and the Los Alamos Laboratory's library made it only reasonable to prepare a somewhat more formal edition. This edition again presented only the translated problems and their contemporary addenda, and has been distributed by the Los Alamos laboratory since then. The recent reconcentration and expansion of interest in the Book, including the 1979 conference, had made a place for a new edition, including a collection of at least some of the work which has been stimulated by the Scottish Book problems in the years since they were first collected.

This project enjoyed the aid of several individuals and institutions, beginning with the encouragement of Stan Ulam and Gian-Carlo Rota. I sincerely thank all of the commentators for their generosity in providing the commentaries and suggestions. It is obvious that a major contributor to this edition is Jan Mycielski. His encouragement and constant flow of comments and references kept life in the project. Bill Beyer provided many significant comments on the formulation of the problems.

The Scottish Book conference, which was held in Denton in May 1979, focused our efforts. It was my hope that some of the spirit of that time and place would be recaptured at the conference. Perhaps, it was through the contributions of the speakers including R. D. Anderson and D. A. Martin, both of whom traced some of the most outstanding work in their fields back to the Scottish Book.

The National Science Foundation, through grant MCS-79-0971 and North Texas State University, provided funding for the conference. A number of commentaries were written under the auspices of a Faculty Research grant from North Texas State University. I sincerely thank Lynn Holick for the superb typing, and the people at Birkhäuser Boston for their help in bringing the project to fruition.

San Diego, CA, USA R. Daniel Mauldin
1981

Preface to the Limited Los Alamos Edition of 1957

The enclosed collection of mathematical problems has its origin in a notebook, which was started in Lwów, in Poland in 1935. If I remember correctly, it was S. Banach who suggested keeping track of some of the problems occupying the group of mathematicians there. The mathematical life was very intense in Lwów. Some of us met practically every day, informally in small groups, at all times of the day to discuss problems of common interest, communicating to each other the latest work and results. Apart from the more official meetings of the local sections of the Mathematical Society (which took place Saturday evenings, almost every week!), there were frequent informal discussions mostly held in one of the coffee houses located near the University building - one of them a coffee house named "Roma," and the other "The Scottish Coffee House." This explains the name of the collection. A large notebook was purchased by Banach and deposited with the headwaiter of the Scottish Coffee House, who, upon demand, would bring it out of some secure hiding place, leave it at the table, and after the guests departed, return it to its secret location.

Many of the problems date from years before 1935. They were discussed a great deal among the persons whose names are included in the text, and then gradually inscribed into the "'book"' in ink. Most of the questions proposed were supposed to have had considerable attention devoted to them before an "'official"' inclusion into the "'book"' was considered. As the reader will see, this general rule could not guarantee against an occasional question to which the answer was quite simple or even trivial.

In several instances, the problems were solved, right on the spot or within a short time, and the answers were inscribed, perhaps some time after the first formulation of the problem under question.

As most readers will realize, the city of Lwów, and with it the "Scottish Book," was fated to have a very stormy history within a few years of the book's inception. A few weeks after the outbreak of World War II, the city was occupied by the Russians. From items at the end of this collection, it is seen that some Russian mathematicians must have visited the town; they left several problems (and prizes for their solutions). The last date figuring in the book is May 31, 1941. Item Number

193 contains a rather cryptic set of numerical results, signed by Steinhaus, dealing with the distribution of the number of matches in a box! After the start of war between Germany and Russia, the city was occupied by German troops that same summer and the inscriptions ceased.

The fate of the Scottish Book during the remaining years of war is not known to me. According to Steinhaus, this document was brought back to the city of Wrocław by Banach's son, now a physician in Poland. (Many of the surviving mathematicians from Lwów continued their work in Wrocłow. The tradition of the Scottish Book continues. Since 1945, new problems have been formulated and inscribed and a new volume is in progress.)

A general word of explanation may be in order here. I left Poland late in 1935 but, before the war, visited Lwów every summer in 1936, '37,'38, and '39. The last visit was during the summer preceding the outbreak of World War II, and I remember just a few days before I left Poland, around August 15, the conversation with Mazur on the likelihood of war. It seems that in general people were expecting another crisis like that of Munich in the preceding year, but were not prepared for the imminent world war. Mazur, in a discussion concerning such possibilities, suddenly said to me " A world war may break out. What shall we do with the Scottish Book and our joint unpublished papers? You are leaving for the United States shortly, and presumably will be safe. In case of a bombardment of the city, I shall put all the manuscripts and the Scottish Book into a case which I shall bury in the ground." We even decided upon a location of this secret hiding place; it was to be near the goal post of a football field outside the city. It is not known to me whether anything of the sort really happened. Apparently, the manuscript of the Scottish Book survived in good enough shape to have a typewritten copy made, which Professor Steinhaus sent to me last year (1956).

The existence of such a collection of problems was mentioned on several occasions, during the last 20 years, to mathematical friends in this country. I have received, since, many requests for copies of this document. It was in answer to such oral and written requests that the present translation was made. This spring in an article, "Can We Grow Geniuses in Science?", which appears in Harper's June 1957 issue, L. L. Whyte alluded to the existence of the Scottish Book. Apparently, the diffusion of this small mystery became somewhat widespread, and this provided another incentive for this translation.

Before deciding to make such an informal distribution, I consulted my teacher and friend (and senior member of the group of authors of the problems), Professor Steinhaus, about the propriety of circulating this collection. With his agreement, I have translated the original text (the original is mostly in Polish) in order to make it available through this private communication.

Even as an author or co-author of some of the problems, I have felt that the only practical and proper thing to do was to translate them verbatim. No explanations or reformulations of the problems have been made.

Many of the problems have since found their solution, some in the form of published papers (I know of some of my own problems, solutions to which were

published in periodicals, among them Problem 17.1, Z. Zahorski, Fund. Math., Vol. 34, pp. 183–245 and Problem 77(a), R. H. Fox, Fund. Math., Vol. 34, pp. 278–287).

The work of following the literature in the several fields with which the problems deal would have been prohibitive for me. The time necessary for supplying the definitions or explanations of terms, all very well understood among mathematicians in Lwów, but perhaps not in current use now, would also be considerable. Some of the authors of the problems are no longer living, and since one could not treat uniformly all the material, I have decided to make no changes whatsoever.

Perhaps, some of the problems will still present an actual interest to mathematicians. At least the collection gives some picture of the interests of a compact mathematical group, an illustration of the mode of their work and thought; and reflects informal features of life in a very vital mathematical center. I should be grateful if the recipients of this collection were willing to point out errors, supply information about solutions to problems, or indicate developments contained in recent literature in topics connected with the subjects discussed in the problems.

It is with great pleasure that I express thanks to Miss Marie Odell for her help in editing the manuscript and to Dr. Milton Wing for looking over the translated manuscript.

Los Alamos, NM, USA S. Ulam
May 1977

Preface to the Limited Los Alamos Edition of 1977 Monograph

Numerous requests for copies of this document, addressed to Los Alamos Scientific Laboratory library or to me, appear to make it worthwhile (after a lapse of some 20 yr) to reprint, with some corrections, this collection of problems.

This project was made possible through the interest and active help of Robert Krohn of the laboratory.

It is a pleasure to give special thanks to Dr. Bill Beyer for his perspicacious review of the changes and the revised version of some formulations. Thanks are due to Martha Lee Delanoy for editorial work.

Los Alamos, NM, USA S. Ulam
May 1977

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