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Resummation and Renormalization in Effective Theories of Particle Physics



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Preface

The elective course titled “Finite-Temperature Quantum Fields” has been part of the curriculum of the master’s degree in physics program at Eötvös University for about 15 years. The original one-semester course was introduced by one of the authors (A.P.) and extended to a two-semester series in 2013 by the second author (A.J.) in cooperation with Dr. Zsolt Szép. The aim of the lecturers was and remains to enable students to acquire the conceptual knowledge and the technical tools of quantum field theory at a level that allows them to participate in research projects of current international interest. That success in this goal has been achieved is indicated by the fact that several of the sections of the present monograph grew out of such joint work carried out with students attending the course.

The material covered in these notes should help to improve students’ ability to deal in general with reorganizations of the perturbation series of renormalizable theories. The specific topics selected reflect the subject area of our own research. From the middle of the 1990s, thermodynamic changes occurring in the ground state of the strong and electroweak vacuum has been at the center of our scientific interest. Although the fundamental theories accounting in principle for all relevant phenomena are well established, field theories with effective degrees of freedom (such as the sigma-meson and constituent quarks) were constructed with an eye to the essential features of the underlying dynamics. These models are able to account for the qualitative changes and even to reproduce some known results semiquantitatively.

It was definitely not our goal to transform our handwritten notes into a monograph when we decided to write a book. We hoped to preserve the characteristics of a university course in which one learns both concepts and “recipes” of quantum field theory mostly by the detailed technical presentation of relevant examples. The eight chapters of this book can be divided into four parts, each of a different character. The historic introductory chapter is followed by two chapters reviewing the basics of quantum field theory necessary for following the directions of contemporary research. The next three chapters introduce three different and equally widely used approaches to improving convergence properties of renormalized perturbation theory. Finally, the last two chapters discuss some physical features of strong and

electroweak matter relying to a large extent on the theoretical tools developed in the previous chapters. Some frequently used or more complicated formulas are collected in three appendices.

Most of the presented material is widely known, which explains the relatively low number of external references. Our citations point either to papers in which specific results of central interest have appeared or to those works that we found (very subjectively) the most instructive. We are grateful to the entire community of theoretical particle physicists investigating characteristic features of strong and electroweak matter, from whom many ideas reflected in the material of the present notes originate.

Nevertheless, we would like to express our specific gratitude to a few colleagues who helped us in our work in the field of the effective models of particle physics in an essential way. For a very fruitful period of our scientific activity we thank to Professors F. Karsch (Bielefeld-Brookhaven) and K. Kajantie (Helsinki). Our research and the present lecture series were shaped in important ways by long-term collaboration with Zsolt Szép and Péter Petreczky. We thank also Szabolcs Borsányi, Tamás Herpay, Dénes Sexty, Péter Kovács, Gergely Markó, and Gergely Fejős.

This book is dedicated to the memory of Professor Péter Szépfalusy (1932–2015). His perfectionist lectures that we attended as students still represent an ideal for us. It was our privilege to have participated in joint research projects with Péter. His results on the application of the $1/N$ expansion to magnetic systems represented a constant source of intuition during our collaboration on the temperature-induced variations of the excitations in the quark–meson theory.

Budapest, Hungary
April 2015

Antal Jakovác
András Patkós

Abbreviations

1PI	One-particle irreducible
2PI	Two-particle irreducible
2PR	Two particle reducible
3D	Three-dimensional
nPI	n -Particle irreducible
BEH-effect	Brout–Englert–Higgs effect
BSM	Beyond standard model
\mathcal{B}	Bubble integral (d dimensions)
\mathcal{B}_3	Bubble integral (3 dimensions)
\mathcal{B}^F	Finite part of the bubble integral (d dimensions)
$\mathcal{B}_d^{(0)}$	Logarithmically divergent part of the bubble integral (d dimensions)
CEP	Critical endpoint
CTP	Closed time path
DR	Dimensional reduction
DS	Dyson–Schwinger
ECCP	Equivalence class of constant physics
EoM	Equation of motion
EoS	Equation of state
FAC	Fastest apparent convergence
FRG	Functional renormalization group
HRG	Hadronic resonance gas
HTL	Hard thermal loop
IR-divergence	Infrared divergence
KMS-condition	Kubo–Martin–Schwinger condition
LCP	Line of constant physics
LO	Leading order
MC simulation	Monte Carlo simulation
MS-scheme	Minimal subtraction scheme
NLO	Next-to-leading order

OMS-scheme	On-mass-shell scheme
OPT	Optimized perturbation theory
PCAC	Partially conserved axial current
PMS	Principle of minimal sensitivity
RS	Resummation scheme
R/A formalism	Retarded/advanced formalism
SB limit	Stefon–Boltzmann limit
SM	Standard model
SSB	Spontaneous symmetry-breaking
QCD	Quantum chromodynamics
QED	Quantum electrodynamics
QGP	Quark gluon plasma
\mathcal{T}	Tadpole integral in d dimensions
\mathcal{T}_3	Tadpole integral in 3 dimensions
\mathcal{T}^F	Finite part of the tadpole integral in d dimensions
$\mathcal{T}_d^{(2)}$	Quadratically divergent part of the tadpole integral in d dimensions
TCP	Tricritical point

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