Spacecraft Momentum Control Systems
Preface

It’s remarkable how often we, the authors, have had a similar experience. At a conference, or during a break at a technical meeting, someone asks “can you recommend a good book about CMGs and reaction wheels?”

The answer is always about the same: “Well, there’s this spacecraft dynamics book and that spacecraft design book, and the new edition of that old reference book we all use, but none of them really talk about momentum control in any depth. You probably already know as much as you’ll find there.” Then there’s a pause. “Not that those books are bad; I’m not saying that. They’re a decent start for a certain audience, such as students who have never worked on a flight program.”

“How about academic articles?”

“Sure, there are a few helpful survey papers and some useful older stuff—especially from the ’70s. For some reason they really seemed to know what they were doing back then.” Another awkward pause. “I don’t really want to go digging into all that.” Then, inevitably, “maybe you should just write a book.”

So that’s what we did.

The reader will find that this book differs from other books on spacecraft dynamics and control. Others provide a broad overview of actuators, sensors, and feedback-control architectures without ever going into these important matters of implementation. And while there exist whole books on propulsive actuators, offering useful depth in the design and operation of rocket engines such as those used for reaction control, there is nothing analogous for momentum actuators. But omitting momentum actuators from a treatment of spacecraft design is like explaining all about automobiles, except for the engine and the transmission. So, finally, there’s a book that addresses the crucial matters of what kind and how many momentum devices to implement, how they should be sized, and how to control the array of them.

This book is an effort to offer a complete picture of momentum actuators—spinning rotors and gimbaled devices—for use in attitude control of spacecraft. It’s a picture that combines our diverse experience in government space systems (satellites for the Air Force, Navy, and NASA) as well as in the commercial space industry and academia. The scope of this book extends from electromechanical
details of individual actuators to space-system architecture issues of interest in spacecraft concept development. We discuss the foundational rigid- and flexible-body dynamics, the subtle mathematics of steering multiple devices within an array, and the applications of these technologies.

These momentum actuators are at the heart of contemporary spacecraft that perform Earth imaging. The rapid growth of commercial success in this application area since the beginning of the twenty-first century is ultimately due to the technological capabilities that these actuators offer. In the decades to come, our industry is likely to see new applications: asteroid mining, in-orbit servicing and repair of satellites, and new human-space missions, all of which will require high torque and momentum storage. Small spacecraft, now the most commonly launched type of satellite, are only just beginning to incorporate sophisticated momentum control, thanks to entrepreneurial investment and a new generation of passionate spacecraft technologists. The momentum devices described in this book enable contemporary spacecraft and will make the future possible.

The authors hope that the breadth of information offered here, most of which has never been collected in one place, will serve the needs of this new generation of spacecraft engineers. And, at least as important, we’ll have an answer to that perennial question, “can you recommend a book about this stuff?”

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Author Biographies

Frederick A. Leve, Research Aerospace Engineer at the Air Force Research Laboratory, Space Vehicles Directorate, received his Ph.D. in Aerospace Engineering from the University of Florida in 2010. While at the University of Florida, he received the IAF Silver Hermann Oberth Medal and the AIAA Abe Zarem Award for Astronautics. Currently, he is the technical advisor to the Guidance, Navigation, and Controls Group and has published many papers in the area of attitude dynamics and control, specifically with respect to momentum control systems and also performs research in under-actuated control, fault tolerant control, control allocation, analytical mechanics, and system identification. Dr. Leve is a recipient of the 2014 AFRL Early Career Award.

Brian J. Hamilton, Engineering Fellow at Honeywell Aerospace, received a BSEE with Honors from the University of Illinois, 1976. Mr. Hamilton has nearly 40 years of experience at Honeywell (formerly, Sperry) and has participated in the development of CMG technology since its infancy. In recent years, his research focus has been on CMG array control and steering, and the general application of momentum systems to agile spacecraft attitude control. Other areas of specialty include nonlinear modeling, controls design, system optimization, and active magnetic suspension. Mr. Hamilton holds 12 patents.

Mason A. Peck, Associate Professor at Cornell University in Mechanical and Aerospace Engineering, received his Ph.D. in Aerospace Engineering from the University of California, Los Angeles, in 2001. He has worked as an aerospace engineer since 1994 and has been on the faculty at Cornell since 2004. From late 2011 through early 2014, he was NASA’s Chief Technologist. In that role, he served as the agency’s chief strategist for technology investment and prioritization and advocate for innovation in aeronautics and space technology. His research lab
focuses on fundamental research in space technology that can be advanced through flight experiments. Examples include Violet, a nanosatellite for demonstrating CMG steering laws, and KickSat, the world’s first crowdfunded spacecraft. Dr. Peck holds 19 patents in the USA and the E.U. and has over 100 academic publications. He received the NASA Distinguished Public Service Medal in 2014.
<table>
<thead>
<tr>
<th>Acronyms</th>
<th>Description</th>
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<tbody>
<tr>
<td>CMG</td>
<td>Control moment gyroscope</td>
</tr>
<tr>
<td>DGCMG</td>
<td>Double-gimbal control moment gyroscope</td>
</tr>
<tr>
<td>IGA</td>
<td>Inner gimbal assembly</td>
</tr>
<tr>
<td>IV</td>
<td>Induced vibration</td>
</tr>
<tr>
<td>LOS</td>
<td>Line-of-sight</td>
</tr>
<tr>
<td>MBS</td>
<td>Mass balancing system</td>
</tr>
<tr>
<td>MCS</td>
<td>Momentum control system</td>
</tr>
<tr>
<td>rms</td>
<td>Root mean square</td>
</tr>
<tr>
<td>rpm</td>
<td>Revolutions per minute</td>
</tr>
<tr>
<td>RWA</td>
<td>Reaction wheel assembly</td>
</tr>
<tr>
<td>SGCMG</td>
<td>Single-gimbal control moment gyroscope</td>
</tr>
<tr>
<td>VSCMG</td>
<td>Variable-speed control moment gyroscope</td>
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Symbols

\( F \)  
Inertial reference frame

\( F_{Gi} \)  
\( i \)th CMG gimbal reference frame

\( F_{Ri} \)  
\( i \)th CMG rotor reference frame

\( A \)  
Arbitrary point in spacecraft reference frame

\( p_i \)  
CMG nominal center of mass reference point at the intersection of the
gimbal and rotor axes assumed fixed in the spacecraft body reference
frame

\( C_B \)  
Spacecraft body center of mass point

\( C_{Gi} \)  
\( i \)th CMG gimbal center of mass point

\( C_{Ri} \)  
\( i \)th CMG rotor center of mass point

\( \rho \)  
Differential mass vector position from center of mass of a component

\( \mathbf{r}_{B/A} \)  
Vector from point \( A \) to spacecraft body center of mass, \( C_B \)

\( \mathbf{r}_{p_i/A} \)  
Vector from point \( A \) to point \( p_i \)

\( \mathbf{r}_{Gi/p_i} \)  
Vector from point \( p_i \) to CMG gimbal center of mass point

\( \mathbf{r}_{Ri/p_i} \)  
Vector from point \( p_i \) to CMG rotor center of mass point

\( \mathbf{b}_i \)  
Spacecraft body coordinate axis “\( i \)”

\( \mathbf{s}_i \)  
\( i \)th CMG spin coordinate axis

\( \mathbf{\hat{o}}_i \)  
\( i \)th CMG output torque coordinate axis

\( \mathbf{\hat{g}}_i \)  
\( i \)th CMG gimbal coordinate axis

\( m_B \)  
Spacecraft bus mass

\( m_{Gi,i} \)  
\( i \)th CMG gimbal mass

\( m_{R,i} \)  
\( i \)th CMG rotor mass

\( m_{Gi,p_i} \)  
\( i \)th CMG gimbal particle imbalance mass

\( m_{R,p_i} \)  
\( i \)th CMG rotor particle imbalance mass

\( \mathbf{J}_s \)  
Spacecraft rigid body inertia dyadic

\( \mathbf{J}_t \)  
Rotor transverse inertia

\( \mathbf{J} \)  
Spacecraft inertia dyadic for the spacecraft’s mass center, including the
spacecraft as a rigid body and all momentum-control devices

\( \mathbf{J}_B \)  
Inertia dyadic of spacecraft about its own center of mass

\( \mathbf{J}_A \)  
Inertia dyadic of spacecraft about point \( A \)
Symbols

$J^G_i$ Inertia dyadic of $i$th CMG gimbal about its own center of mass

$J^G_A$ Inertia dyadic of $i$th CMG gimbal about point $A$

$J^R_i$ Inertia dyadic of $i$th CMG rotor about its own center of mass

$J^R_A$ Inertia dyadic of $i$th CMG rotor inertia about point $A$

$J_{gr,i}$ Scalar inertia component of $i$th CMG gimbal-wheel assembly about its gimbal axis

$J_{g,i}$ IGA inertia dyadic, i.e. everything that contributes to the IGA rigid-body inertia in all axes

$J_{g,eff}$ Effective scalar IGA inertia. This inertia includes everything rigid that the gimbal must accelerate along with the output-axis stiffness ($K_{OA}$) effect: $J_{g,eff} = J_g + \frac{h^2}{K_{OA}}$

$J_{g,eff}$ Effective IGA inertia dyadic. This inertia includes everything that contributes to the IGA rigid-body inertia in all axes, along with the output-axis stiffness effect along the gimbal axis direction.

$h^B_A$ Angular momentum of spacecraft bus about point $A$

$h^B_B$ Angular momentum of spacecraft bus about its own center of mass

$h^i$ Angular momentum of $i$th CMG gimbal about $p_i$

$h^G_i$ Angular momentum of $i$th CMG gimbal about its own center of mass

$h^R_i$ Angular momentum of $i$th CMG rotor about $p_i$

$h^R_i$ Angular momentum of $i$th CMG rotor about its own center of mass

$h_r$ Angular momentum vector of a single CMG rotor

$h$ Angular momentum body coordinate matrix representation for an array of CMG or RWA

$h_s$ Superspin

$\tau_{gf,i}$ Internal friction torque of $i$th CMG gimbal

$\tau_{rf,i}$ Internal friction torque of $i$th CMG rotor

$\tau_{r,i}$ $i$th scalar rotor torque for a single-gimbal CMG (for a perfectly aligned and rigid rotor)

$\tau_{g,i}$ $i$th scalar gimbal torque for a single-gimbal CMG (for a perfectly aligned and rigid IGA and gimbal)

$\tau_d$ Scalar drag torque

$\tau_o$ Vector output torque for a single-gimbal CMG (for a perfectly aligned and rigid IGA and gimbal). This torque acts on the spacecraft and is therefore equal in magnitude but opposite in direction to the torque that acts on the CMG.

$\omega_B^B/N$ Angular acceleration vector of a spacecraft-fixed reference frame $B$ relative to an inertial frame $N$. The overdot associated with a scalar derivative has been replaced with the letter $B$ to indicate that the derivative has been taken relative to the $B$ reference frame
\( \omega^{B/N} \) Angular velocity vector of spacecraft body with respect to the inertial reference frame

\( \omega^{G_i/B} \) Angular velocity vector of \( i \)th CMG gimbal with respect to the spacecraft body reference frame

\( \omega^{R_i/G_i} \) Angular velocity vector of \( i \)th CMG rotor with respect to the \( i \)th CMG gimbal reference frame

\( \omega \) Scalar spacecraft angular rate or body coordinates matrix representation of spacecraft angular velocity (context sensitive)

\( v_A \) Translational inertial velocity of point \( A \)

\( \Omega_{r,i} \) \( i \)th scalar CMG rotor rate

\( \Omega_r, \Omega_r \) Matrices of CMG rotor spin rates and accelerations

\( \Delta, \dot{\Delta}, \ddot{\Delta} \) Matrices of CMG gimbal angles, rates, and accelerations

\( \delta_i, \dot{\delta}_i, \ddot{\delta}_i \) \( i \)th CMG gimbal angle, rate, and acceleration

\( \beta \) CMG array skew angle

\( \gamma \) CMG array clocking angle

\( K_{OA} \) Output axis stiffness

\( K_d \) Performance ratio (maximum rate over maximum acceleration)

\( K_c \) Performance ratio (maximum acceleration over maximum jerk)

\( K_T \) Motor torque constant (torque per unit current)

\( K_{MD} \) Motor \( K_M \) density (ft-lb/sqrt(W) per lb)

\( i \) Index for a single CMG within a multiple-CMG array

\( n \) Number of CMGs in a multiple-CMG array

\( m \) Singularity measure

\( \alpha \) Singularity parameter and scalar angular acceleration (context sensitive)

\( j \) Scalar angular jerk (context sensitive)

\( B^Q_A \) Direction-cosine matrix that relates the representation of a vector \( v \) in \( B \) coordinates \((B^v)\) to its representation in \( A \) coordinates \((A^v)\)