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Ping Zhang

Color-Induced Graph Colorings

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Ping Zhang
Department of Mathematics
Western Michigan University
Kalamazoo, MI, USA

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Preface

The interest in edge colorings of graphs can be traced back to 1880 when the Scottish mathematician Peter Guthrie Tait attempted to solve the Four Color Problem with the aid of edge colorings. Despite the fact that Tait's approach was not successful, it initiated a new concept. In 1964, Vadim Vizing proved that the minimum number of colors needed to color the edges of a graph so that every two adjacent edges are colored differently (proper edge colorings) is one of two numbers, namely either the maximum degree or the maximum degree plus one. This result led to an increased interest and study of edge colorings in graph theory, not only edge colorings that are proper but also edge colorings that are not.

In recent decades, there has been great interest in edge colorings that give rise to vertex colorings in a variety of ways, which is the subject of this book. While we will be describing many ways in which edge colorings have induced vertex colorings and some of the major results, problems, and conjectures that have arisen in this area of study, it is not our goal to give a detailed survey of these subjects. Indeed, it is our intention to provide an organized summary of several recent coloring concepts and topics that belong to this area of study, with the hope that this may suggest new avenues of research topics.

In Chap. 1, we begin with a brief review of the well-known concepts of proper edge colorings and proper vertex colorings, including many fundamental results concerning them.

In Chap. 2, unrestricted edge colorings of graphs are considered whose colors are elements of the set $\mathbb{N}$ of positive integers or a set $[k] = \{1, 2, \ldots, k\}$ for some positive integer k. From such an edge coloring c of a graph G, a sum-defined vertex coloring c' is defined, that is, for each vertex v of G, the color $c'(v)$ of v is the sum of the colors of the edges incident with v. The edge coloring c is vertex-distinguishing or irregular if the resulting vertex coloring c' has the property that $c'(u) \neq c'(v)$ for every pair u, v of distinct vertices of G. The minimum positive integer k for which a graph G has such a vertex-distinguishing edge coloring is the irregularity strength of G. In Chap. 3, the corresponding coloring is considered in which the colors are taken from a set $\mathbb{Z}_k$ of integers modulo k.

Chapter 4 also deals with unrestricted edge colorings c of graphs but here the induced vertex coloring is defined so that the color $c'(v)$ of a vertex v is the set of colors of its incident edges. In Chap. 5, the emphasis changes from vertex colorings that are set-defined to those that are multiset-defined. In both cases, the induced vertex colorings c' are vertex-distinguishing.

In Chap. 6, unrestricted edge colorings c of graphs are once again considered but in this case the induced vertex colorings c' are neighbor-distinguishing, that is, $c'(u) \neq c'(v)$ for every two adjacent vertices u and v. In this chapter, two vertex colorings c are defined, both where the colors belong to a set $[k]$, one where $c'(v)$ is sum-defined and the other where $c'(v)$ is multiset-defined. Chapter 7 is devoted to unrestricted edge colorings of graphs whose colors are elements of $\mathbb{Z}_k$ of integers modulo k that induce a sum-defined, neighbor-distinguishing vertex coloring.

In Chap. 8, both proper and unrestricted edge colorings are considered, and the vertex colorings are set-defined, using elements of $[k]$ as colors. In Chap. 9, the edge colorings are proper and the vertex colorings considered are sum-defined, using elements of $[k]$ as colors. In these two chapters, the properties of being vertex-distinguishing and neighbor-distinguishing are both described. Chapter 9 ends with a discussion of so-called twin edge colorings, which are proper edge colorings that use the elements of $\mathbb{Z}_k$ as colors and that induce proper vertex colorings that are sum-defined.

The following table summarizes all types of edge colorings considered in this book and the resulting vertex colorings. In particular, the table describes, in each chapter:

1. the condition placed on the edge coloring,
2. the sets from which the edge colors are selected,
3. the definition of the vertex colors, and
4. the property required of the resulting vertex coloring.

Chapter 1: Introduction
Chapter 2: The Irregularity Strength of a Graph
Unrestricted Edge Colorings, $\mathbb{N}$, Sum-defined, Vertex-Distinguishing.
Chapter 3: Modular Sum-defined, Irregular Colorings
Unrestricted Edge Colorings, $\mathbb{Z}_k$, Sum-defined, Vertex-Distinguishing.
Chapter 4: Set-Defined Irregular Colorings
Unrestricted Edge Colorings, $\mathbb{N}$, Set-defined, Vertex-Distinguishing.
Chapter 5: Multiset-Defined Irregular Colorings
Unrestricted Edge Colorings, $\mathbb{N}$, Multiset-defined, Vertex-Distinguishing.
Chapter 6: Sum-Defined Neighbor-Distinguishing Colorings
Unrestricted Edge Colorings, $\mathbb{N}$, Sum-defined, Neighbor-Distinguishing.
Chapter 7: Modular Sum-Defined Neighbor-Distinguishing Colorings
Unrestricted Edge Colorings, $\mathbb{Z}_k$, Sum-defined, Neighbor-Distinguishing.

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Chapter 9: Sum-Defined Colorings by Proper Edge Colorings
9.1. Proper Edge Colorings, $\mathbb{N}$, Sum-defined, Vertex-Distinguishing.
9.2. Proper Edge Colorings, $\mathbb{N}$, Sum-defined, Neighbor-Distinguishing.
9.3. Proper Edge Colorings, $\mathbb{Z}_k$, Sum-defined, Neighbor-Distinguishing.

Kalamazoo, MI, USA Ping Zhang

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