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Kimball A. Milton

Schwinger's Quantum Action Principle

From Dirac's Formulation Through
Feynman's Path Integrals,
the Schwinger-Keldysh Method,
Quantum Field Theory, to Source Theory

 Springer

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Abstract

Starting from the earlier notions of stationary action principles, we show how Julian Schwinger's Quantum Action Principle descended from Dirac's formulation, which independently led Feynman to his path-integral formulation of quantum mechanics. The connection between the two is brought out, and applications are discussed. The Keldysh-Schwinger time-cycle method of extracting matrix elements in nonequilibrium situations is described. The variational formulation of quantum field theory and the development of source theory constitute the latter part of this work. In this document, derived from Schwinger's lectures over four decades, the continuity of concepts, such as that of Green's functions, becomes apparent.

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