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Volume 50

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# Contact Mechanics of Articular Cartilage Layers

## Asymptotic Models

Springer

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ISSN 1869-8433 ISSN 1869-8441 (electronic)
Advanced Structured Materials
ISBN 978-3-319-20082-8 ISBN 978-3-319-20083-5 (eBook)
DOI 10.1007/978-3-319-20083-5

Library of Congress Control Number: 2015941864

Springer Cham Heidelberg New York Dordrecht London


Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media
(www.springer.com)

*To Alina and Wiktoria*

# Preface

This book is based on research results developed during our Marie Curie project devoted to asymptotic modeling of articular contact. Over the past two decades, articular contact mechanics has developed significantly in response to the increasing demand from orthopedics. While significant progress has been achieved in the mathematical modeling of the contact interaction between articular cartilage layers, we realized when writing the plan of the book that many of those results, obtained in the isotropic case, could be generalized for the case of transverse isotropy. The latter is of paramount importance in contact biomechanics, because many biological tissues like articular cartilage should be modeled as transversely isotropic material. Correspondingly, this plan required that many of the lacunae found in existing theory be filled in and allowed us to correct some misprints and omissions. Overall, our aim was to create a compendium of knowledge on asymptotic models for contact interactions of thin elastic/viscoelastic/biphasic layers.

Generally speaking, this book is about mathematical models for unilateral contact problems (with a priori unknown contact area) involving thin linearly deformable layers bonded to rigid bodies (called substrates) which transfer an externally applied compressive load. Particular attention is paid to analysis of the contact pressure distribution between the layers as well as to the relation between the contact force and the contact approach between the substrates.

From a methodological point of view, the book implements a unified asymptotic approach focused on deriving approximate analytical solutions, which are presented in the form of simplified mathematical models, called asymptotic models. Though we make use of rigorous mathematical methods, we tried to avoid excessive technical details of the asymptotic analysis in order to simplify the presentation of the book's material for a broader audience. With the same purpose we adopt unified notation across the different chapters.

The asymptotic technique we employ to model the deformation response of thin layers is generally known in the literature as a perturbation algorithm originally credited to Gol'denveizer [1]. The development of asymptotic methods in contact problems for thin elastic layers originates in the works of Aleksandrov [2], Koiter [3], Alblas and Kuipers [4], and others, although the majority of research papers were devoted to the two-dimensional and axisymmetric cases. The application of asymptotic methods in articular contact problems was initiated by Ateshian et al. [5] and later developed by Wu et al. [6–8] and others.

In principle, analytical solutions are widely viewed as benchmark solutions for numerical methods. At the same time, asymptotic models have an importance of their own: they not only provide insights into the qualitative behavior of numerical solutions to the multiparametric problem under consideration, but also can expand our understanding of more realistic simulation models.

This book is organized into nine chapters. Each chapter is self-consistent, can be read independently, and is supplied with a comprehensive reference list. Chapters 1–4 present asymptotic analysis of the frictionless unilateral contact problems for thin elastic and viscoelastic layers in a form accessible to a general engineering audience. A specific character of deformation models for articular cartilage is discussed in Chap. 5, where a linear biphasic theory is outlined in detail. In Chap. 6, we generalize the asymptotic deformation model of Ateshian et al. [5] for the case of transverse isotropy and study the contact problems for thin bonded biphasic layers. Asymptotic modeling methodology for tibio-femoral contact is presented in Chap. 7, based on the asymptotic models constructed in the previous chapters. Some features reflecting the real structure of articular cartilage such as inhomogeneity are considered in Chap. 8, again from a general point of view and under a simplifying assumption of elastic deformation behavior. Finally, some sensitivity analysis issues for the asymptotic models of articular contact are addressed in Chap. 9.

This monograph is recommended for biomechanics researchers dealing with different aspects of articular contact as well as for Ph.D. students enrolled in contact mechanics and biomechanics courses.

## Acknowledgments

The authors would like to thank the Department of Mathematics at Aberystwyth University for providing excellent academic facilities throughout the duration of the project project that eventually led to this book. The support of the European Commission via the grants FP7, PEOPLE, Marie Curie IE project “OA AM” No. 253055 (2010–2012) and FP7, PEOPLE, Marie Curie IRSES project “TAMER” No. 610547 (2014–2018) is gratefully acknowledged. We also thank Miss Nina Hämäläinen for preparation of the figures and Mr. Oliver Bain for proofreading the manuscript.

Aberystwyth
April 2015

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